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Social security trust fund (SSTF), the government fiscal use of the SSTF, and intergenerational equity

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Social security trust fund (SSTF), the government fiscal use of the SSTF.

and intergenerational equity

by

Jae Kyeong Kim

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CHAPTER 1. INTRODUCTION

A social security (SS) system can be designed in various ways and have its own distinctive purpose to achieve, although this purpose may come in many guises. For example, the system may be designed for the purpose of meeting individuals' life-time utility maximization motives and/or a benevolent reformer's generational welfare maximization motive. As time passes by and the state of economy changes, however, the once established design may not be able to serve its purpose any more. This may be the beginning of the time when the soundness of a SS system is questioned and so is its prospect. The eventual next step would be to take advantage of changes in the economic state to revise the SS system, perhaps with a different purpose.

Although there is no shortage of historical examples for this process, the current trend in SS systems around the world is a case in point. In the years of the post-war population increase and unprecedentally strong economic performance, most developed countries implemented unfunded (pay-as-you-go) SS systems. In recent years, many of these same countries began to move toward a funded SS system through the accumulation of a social security trust fund (SSTF).

It is often considered that the underlying motivations behind the current transformations of SS systems from unfunded to funded could, to a large extent, be accounted for by a wide range of demographic changes, one of which is a decreasing working population relative to the retired population. A typical argument for this view

would look like this: an increasing "dependency ratio," defined as the ratio of retired population to working population, may come against a pure pay-as-you-go SS system because it depresses the rate of return on SS tax contributions.[1] Or, in a bit stronger version, a funded SS system may be preferred to an unfunded one in the presence of the inevitably lower rate of return on pay-as-you-go SS tax contributions. Miguel-Angel and Lopez-Garcia (1991) imply that there is no certain a priori ground for this conclusion, however. They argue that, when the market interest rate is greater than the growth rate of the population, an increase in the population growth rate could possibly improve steady-state welfare in the presence of an unfunded SS system.

Economists have also argued that a transition from an unfunded SS system to a fully funded SS system does not necessarily enhance economic efficiency or lead to welfare improvement. For instance, Breyer (1989) argues that the transition does not compensate the welfare loss of the then-old generation without hurting at least one of the later generations. Homburg (1990) argues against this result. His study suggests that, if labor supply is endogenous, compensating for all the lost SS benefits of the then-old generation through huge amounts of one-period external government bonds would reduce the distortional effect in the labor market and thus the transition from an unfunded to a funded system can be Pareto improving.

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[1] As shown later, the SS tax contribution of an individual agent earns a rate of return which is composed of the rates of growth in generational population and wage. See Aaron (1966) for a more detailed account of the rate of return on SS tax contribution.

Although scenarios of the transition from an unfunded SS system to a fully funded SS system have been presented in several simulation models, the existence of a transition path which would improve the welfare of *every* generation has not been demonstrated.[2] For example, Huang, İmrohoroğlu, and Sargent (1995) evaluate two alternative schemes for compensating losses resulting from the sudden termination of a pay-as-you-go (unfunded) SS system. In the first compensated 'buy out' scheme, the termination of an unfunded SS system is followed by replacing the current SS benefit by a one period government external debt. In the second 'government run' scheme, instead of issuing an 'entitlement' bond, fiscal policies are implemented in such a fashion that the SS benefits are financed through government claims on publicly held private physical capital. The authors show that the second experiment provides larger benefits to later generations, but the efficacy of the 'government run' scheme depends on the performance of private capital.

However, there is an important consideration missing in recent advances in the transition study of SS systems. Most transition studies have focused on *instantaneous* transition from unfunded to funded systems. More realistically, the transition will be gradual, and the key to understanding the transition will lie in the analysis of a partially funded SS system with a varying size of the SSTF.

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[2] For example, see Seidman (1986) for the transition and Auerbach and Kotlikoff (1985) for reversed transition.

The main concern of this dissertation is to try to understand the transformation process of a SS system from unfunded to funded, not just from a comparison of *rates*, such as the rates of return on physical capital and SS tax contributions and the growth rate of the population, and not just from a comparison of two polar SS systems such as unfunded and fully funded, but from a general equilibrium analysis which we believe to be more adequate. We consider a wide range of possible SS arrangements and possible government uses of the SSTF. We have, throughout, a twofold objective. First, we wish to track down the general equilibrium effects of alternative SS arrangements ranging from pure pay-as-you-go to fully funded, including effects on intergenerational equity over time. Second, we wish to explore the various consequences that result from different uses of the SSTF by government under each of the alternative arrangements for which at least some degree of funding occurs.

To carry out this investigation, we first need to construct a basic economic model. We modify the model of an economy developed by Diamond (1965)---hereafter referred to as the "Diamond Economy"--- to incorporate a SS structure. By a Diamond Economy we mean a two period lived overlapping generations economy in which private agents engage in consumption and production decisions. More precisely, in the Diamond Economy there is a single non-storable output (such as a seed that would rot away unless eaten or planted), and each young agent has one unit of labor endowment which is supplied inelastically to a production process in return for a real wage. The wage income may either be consumed when young or invested as physical capital into the subsequent

period's production process in return for capital income which is consumed when old. All agents are structurally identical apart from time of birth. Young agents cannot borrow and old agents do not work. The Diamond Economy is deterministic in the sense that there is no stochastic element. One deviation of our model from the Diamond Economy is that physical capital in any given period depreciates completely at the end of the period.

The SS system is incorporated into the Diamond Economy in parameterized form, which permits the comparative dynamic study of a range of SS systems from pure pay-as-you-go to fully funded. This leads us to review various important historical examples of the ways in which both social and private security systems have been implemented. The current U.S. SS system provides a basic benchmark example. The U.S. system requires a certain amount of SS tax payments saved or dissaved in order to keep a *close actuarial balance* of SS tax payments and benefits during a certain time period.[3] A crucial element of the U.S. SS system relevant to our study lies in the changing size of the SSTF over time, which leads us to realize that the transformation of a SS system from nonfunded to fully funded is capable of being stated in terms of a SSTF fraction, i.e., the fraction of SS tax benefits that are allocated to a SSTF. We therefore consider, in our model, a family of SS arrangements parameterized by two basic exogenously given parameters ranging from 0 to 1: namely, a SS tax rate and a trust fund fraction.

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[3] See "The 1996 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance (1996)" for a detailed definition of "close actuarial balance".

In exploring general equilibrium economic and welfare effects of the SSTF in a more fruitful fashion, we also consider the government use of the SSTF. Contrasted with the traditional assumption in the related literature of the SSTF being invested only as a physical capital item into a production process[4], we postulate that the SSTF could, additionally, take the form either of a redistributive transfer item to current generations or of a human capital investment item resulting in increases in the efficiency of raw labor in next period. The inspiration for this extension again derives from a real world situation, one of which can be traced down in the U.S. SS system, although there is no explicit consideration of that.

That is, in the real world, the SSTF could be different from a private trust fund. While it is fair enough to consider a corporate trust fund as a kind of debt obligation to the firm, it is not always likely to be the case that the SSTF should be thought of as a debt obligation to SS tax payers.[5] For instance, the U.S. government, in a document entitled "Analytical Perspectives," seems to clarify this matter. "*Unlike the assets of private pension plans, they (U.S. social security trust funds) do not consist of real economic assets that can be drawn down in the future to fund benefits.*" It continues, "*...the Federal Government owns the assets and earnings of Federal trust funds, and it can raise or lower future trust fund collections and payments, or change the purpose for which the*

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[4] This traditional assumption regarding the SSTF is not necessarily accurate for real economies. Only a small number of SS systems in the world satisfy the assumption. One is the Chilean SS system where the SSTF is in the form of mandatory *personal* accounts invested in financial markets. See Diamond and Valdes-Prieto (1994) for a detailed description of the Chilean SS system.

[5] For details, see Bikhchandani and Huang (1994) for Treasury Securities in general. Moreover, see Tabellini (1991) for the difference between SSTF and a public-issued bond.

collections are used, by changing existing law." [6] Strictly following this statement, the

U.S. SSTF is *owned* by the U.S. government, not by SS tax payers. It may surprise some

U.S. SS tax payers who presume that the SSTF is held in the form of an equity claim.

The important implication of the U.S. practice regarding the SSTF for our study is

that the real and ultimate economic and welfare import of the SSTF may depend strongly

on the government's use of the SSTF. The government fiscal policy regarding the SSTF

is parameterized in our model by introducing two additional exogenously given

parameters ranging from 0 to 1: namely, the government redistributive transfer fraction

and the physical capital investment fraction, with any remaining funds then assumed to

be allocated to human capital investment. We then show that varying fiscal policy

regarding the use of the SSTF dramatically alters the response of both private and

government saving, affecting the real wage rate, the rate of return on physical capital, and

the implicit rate of return on SS tax payments. These effects, in turn, have numerous

general equilibrium feedback impacts on private saving. In particular, the capital

accumulation process depends on what the government does with the SSTF.

In short, we incorporate a wide range of possible SS arrangements and possible

government uses of the SSTF into the Diamond Economy. No profound theory will be

found in this study. We have aimed only at putting together some aspects which are

inspired by what we hope is relevant to our study purpose. Private agents pay SS taxes

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[6] Executive Office of the President. Office of Management and Budget Analytical Perspectives, Budget of the United States Government, Fiscal Year 1996. Washington, D.C., U.S. Government Printing Office. 1995, p. 251.

when young and are entitled to receive SS benefits when old. SS benefits are determined, first, by the way the SS system is arranged, and, second, by the way the government uses the SSTF, if any. An agent behaves optimally in terms of his personal saving. His optimizing behavior is summarized by maximizing his lifetime utility subject to a budget constraint, taking as given relative prices and the SS system. The latter is characterized in terms of two SS arrangement parameters and two government fiscal policy parameters regarding the use of the SSTF.

It is worthwhile to note the assumption that young agents have no access to borrowing. This assumption is motivated by the very construction of our economic model. First, there is no way for young agents to engage in purely private borrowing contracts; agents when old have no incentive to lend their old age income to young agents in a given period because there is no way for the young to repay them. Moreover, the model does not postulate the existence of an unbacked bond market either by the government as assumed in the later part of Diamond (1965) or by a private financial intermediary as assumed in Pingle and Tesfatsion (1997).

After constructing and analyzing our economic model, we simulate the model to explore the general equilibrium economic and intergenerational equity consequences of introducing alternative SS systems. As argued in Kydland and Prescott (1996), in

simulation experiments it is important to start by posing a set *of well-defined quantitative questions.*[7] We primarily focus on the following questions:

1. *How will the model economy respond to different SS arrangements, from unfunded to funded, and to different uses by government of the SSTF?*

2. *How effective is the SS actuarial status of individual agents (i.e., benefits received relative to taxes paid, in present value terms) as a measure of intergenerational equity? What could be an alternative measure?*

3. *What are the consequences of alternative SS arrangements and government uses of the SSTF for intergenerational equity measures in terms of the time profiles of SS actuarial status and an alternatively postulated measure based on normalized lifetime utility? For instance, does a build-up of the SSTF necessarily improve intergenerational equity in terms of either of these two measures?*

4. *In general, does there exist any meaningful relationship(s) between the size and use of the SSTF and intergenerational equity? If so, how does it depend on structural aspects of the economic such as the population size, labor share, and consumer time preference?*

Some of the results found during the course of the simulations are surprising. First, there are several occasions in which the specific type of SS arrangement and of the government policy use of the SSTF does not matter, as will be clarified later on. The

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[7] Following Kydland and Prescott (1996), one of the best examples can be found in Auerbach and Kotlikoff (1987).

reason for this is that what is ultimately relevant for agent's rational economic decision making is whether or not his lifetime utility increases, not the particular way of earning income or the timing of income. In short, as long as their lifetime utilities remain unchanged, agents do not distinguish between types of SS systems.

Second, SS actuarial status is not an effective measure of intergenerational equity simply because the measure is not sufficient to reflect the full gain or loss of agent utility associated with a switch between SS systems. This suggests how an evaluation of a SS arrangement strictly on the basis of the actuarial status of consumers can be mistaken.

Third, the intergenerational welfare consequences of the government policy use of the SSTF depends not only on the type of government expenditure, more particularly, on who will receive the benefits, but also on the extent of the benefits. For instance, it is mistaken to assume that future generations will necessarily be free from possible burden in the case of a financial squeeze simply because the SSTF is spent for their sake in the form of human capital investment on their raw labor to enhance their labor productivity. When the government uses the SSTF for human capital investment and the productivity of human capital investment is relatively low, simulations show that increased human capital investment can actually decrease intergenerational equity for *all* generations.

Fourth, surprisingly, the general notion that the population growth rate has an important intergenerational welfare implication regarding a SS system is not necessarily true in our economic model. Our sensitivity tests show that a change in the population growth rate does not always change the ordering of the steady state intergenerational

equity measures over different SS funding policies, indicating that having a particular population growth rate does not necessarily make one type of SS arrangement more preferable than another.

Although there has been a large volume of economic research on the SSTF and intergenerational equity, we consider most of the literature as rather remote to our study because there are many ways in which the economic model of this paper differs from those of the related literature. First, our parameterization of the SS system permits the consideration of a wide range of its possible arrangements. Having a wide range of SS systems discussed in one setting is relatively new to the current economic research in this area. As one of the closest studies to ours, Blanchet and Kessler (1991) explore some optimal funding policies for a SS system by introducing two polar SS systems---a fully funded and an unfunded SS system---into the economy simultaneously. However, they do not consider intermediate cases as is done in the current paper.

The present study differs, again, in that it incorporates the government fiscal policy use of the SSTF. Although critical in evaluating the economic and intergenerational equity effects of a given (partially or fully) funded SS system, a *formal* incorporation of the government use of the SSTF has previously been to a large extent neglected. This neglect can be attributed, partly, to the fact that a large build-up of the SSTF is a relatively new phenomenon, and, partly, to the notion ingrained in the study of SS systems that a consideration of the government use of the SSTF is outside the study of the system. With these two facts intermingled, a certain narrowly defined physical capital

investment use of the SSTF is often assumed even in a general equilibrium analysis. Instead, the current dissertation considers various possible government policy uses of the SSTF including redistributive transfer and human capital investment as well as physical capital investment.

The present study differs, also, in the ways in which it explicitly defines the term "intergenerational equity." As the debate on equity in intergenerational contexts attracts growing attention, the term "intergenerational equity" is used more frequently but often without any concrete quantitative definition of the term. We define two alternative measures of "intergenerational equity." One considered definition is from a "within SS system" perspective, and the other is from a lifetime utility perspective.

The construction of the dissertation is as follows: Chapter 2 sets out the basic model, a Diamond (1965) overlapping generations economy modified to include a SS system. Some of the main dynamic features of the economic model are also analytically derived and discussed. Chapter 3 first raises key questions to be answered during the course of the simulation experiments. The parameterization and computational details of the experiments are then presented. Finally, simulation experiments, including sensitivity tests, are conducted and their results are discussed, answering the key questions. Chapter 4 summarizes the simulation results. Concluding remarks are given in Chapter 5.

CHAPTER 2. THE ECONOMIC MODEL

2.1. Basic Model Structure

The model extends the overlapping generations model first developed by Samuelson (1958) and later by Diamond (1965), by incorporating a social security (SS) system. At the beginning of each period $t \geq 1$, a new generation of agents appears, where each agent lives for two periods. Generation t is the set of all agents born at the beginning of period t. In each period t, the generation t agents are called the young age agents and the generation $t - 1$ agents the old age agents. All agents in this economy are structurally identical apart from time of birth.

Let N_t denote the number of agents in generation t. The population grows at a constant net rate n per period, where $n > -1$. It follows that the law of motion for births is stated as

$$N_{t+1} = (1 + n)N_t, \qquad t \geq 0, \tag{2-1}$$

where the number of old age agents in period 1, N_0, is assumed to be given by some positive constant $\overline{N}_0$.

This basic model structure is depicted in Figure 2.1.

$$
\begin{array}{cccc}
\text{time t - 1} & \text{time t} & \text{time t + 1} & \text{time t + 2}
\end{array}
$$
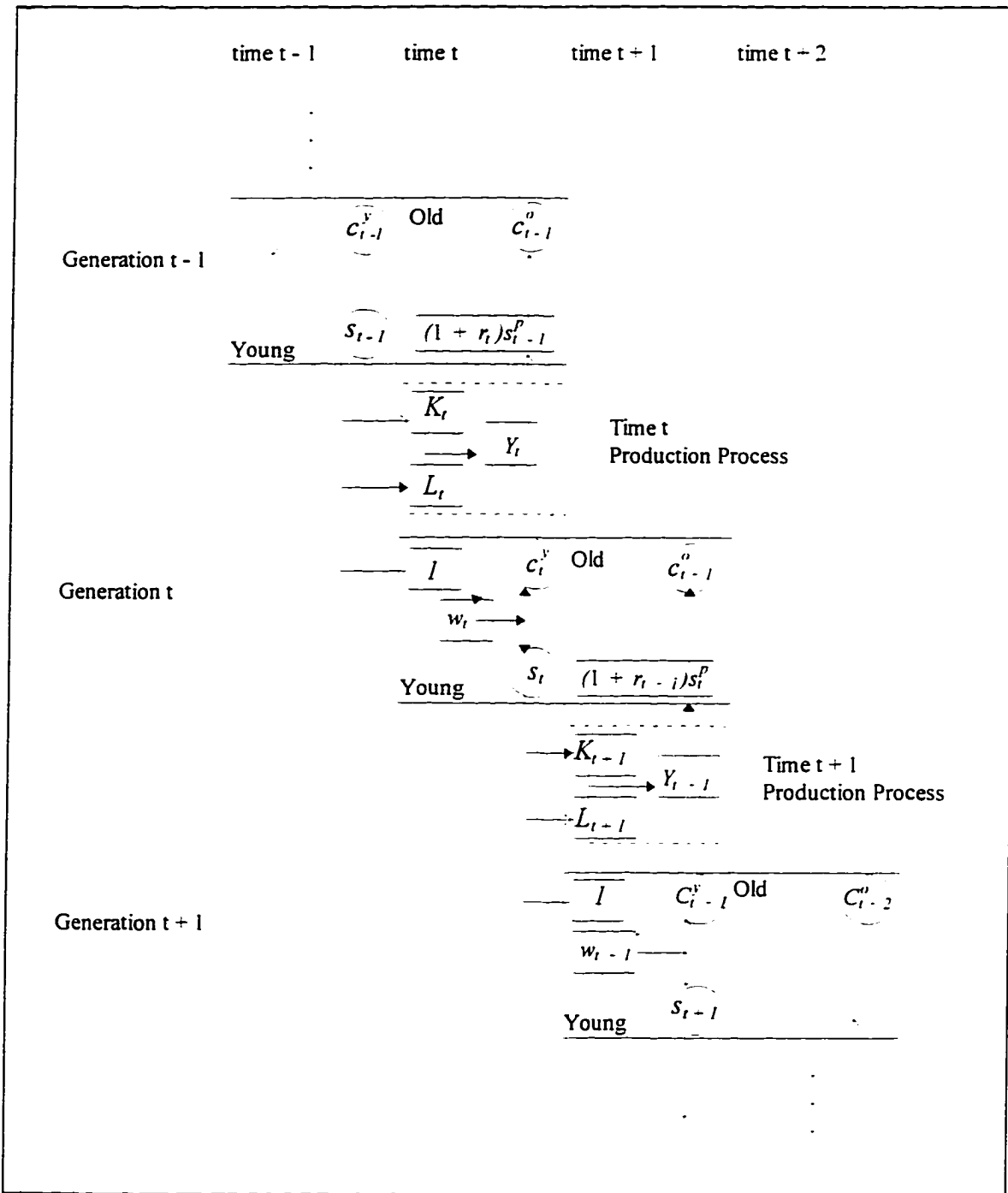
Figure 2.1: The Diamond economy from the perspective of generation t

2.2. Endowments and Preferences

Each young agent in generation $t \geq 1$ works for one period and then retires for the next period. He also has an endowment of time normalized to one at the beginning period of his life which he supplies inelastically to a period t production process for wage income in period t. The only way for the generation t young agent to transfer the wage income in period t to the future period is through the saving of his wage income. This saving is invested in a time $t + 1$ production process in the form of time $t + 1$ physical capital, which generates capital income in period $t + 1$.

A generation t young agent born at the beginning of period t has the following lifetime utility function in period t:

$$U(c_t^y, c_{t-1}^o) = \beta \log c_t^y + (1 - \beta) \log c_{t-1}^o, \tag{2-2}$$

where $U(\bullet, \bullet)$ is the lifetime utility of the generation t young agent; c_t^y and c_{t-1}^o are the consumption levels of the generation t agent in his young and old age, respectively; β is his subjective weight on young age consumption, $0 < \beta < 1$; and $U(\bullet, \bullet)$ is continuous and twice continuously differentiable, strictly increasing in each of its arguments, and strictly concave.

2.3. Budget Constraints

A generation t young agent consumes, saves, and pays a SS tax out of his wage income and lump-sum transfer in period t. Let w_t denote his wage rate in period t in terms of the time t output good and let s_t^p denote his personal saving in period t. The budget constraint faced by the generation t young agent in period t is

$$c_t^y + s_t^p = [1 - \tau]w_t + T_t, \tag{2-3}$$

where τ is the SS tax rate in period t and T_t is a lump-sum transfer in period t.

In period $t + 1$ the generation t old agent will consume his capital income (the return on his saving), SS benefits, b_{t+1}, and lump-sum transfer, T_{t+1}. Let r_{t+1} denote the net rate of return on personal saving in period $t + 1$. Then the budget constraint faced by the generation t old agent in period $t + 1$ is

$$c_{t+1}^o = (1 + r_{t+1})s_t^p + b_{t+1} + T_{t+1}. \tag{2-4}$$

We assume that the generation t young agent has no access to borrowing. This assumption is motivated by the very construction of the model itself. First, there is no incentive for agents to engage in purely private borrowing contracts. The generation $t - 1$ old agents have no incentive to lend their old age income to the generation t young

agents because there is no way for the young to repay them. and vice versa. Second. the

model does not postulate the existence of an unbacked bond market. Thus, the generation

t young consumer faces a restricted access to future possible incomes such as his own

capital income, social security benefit, and lump-sum transfer in period $t + 1$. The

assumption that no one in the economy is allowed to borrow can be stated as

$$s_t^p \geq 0, \qquad \forall t. \tag{2-5}$$

2.4. Social Security System

Suppose that, at the beginning of period 1, a SS system is implemented as an

additional means of securing both generation young and old needs. The basic structure of

the SS system is as follows: Each generation t young agent pays a SS tax in period t

which is proportional to his wage income, and is then entitled to receive a SS benefit

when old. Let τ denote the SS tax rate. The SS tax payment, v_t, is then given by

$$v_t = \tau w_t, \tag{2-6}$$

where $0 \leq \tau \leq 1$.

The government puts aside a fractional amount of the aggregate SS tax payments

in period t in the form of a social security trust fund (SSTF) in period t and distributes the

other fractional amount to the generation t - 1 old agents as part of their social security

benefits in period t. Let δ denote the fraction put aside in the form of an SSTF. The SSTF

in period t, denoted by $SSTF_t$, is then given by

$$SSTF_t = \delta v_t N_t, \tag{2-7}$$

where $0 \le \delta \le 1$.

The use of the SSTF depends on the government fiscal policy. The government

fiscal policy is assumed here to consist of government transfers and government saving,

where the latter in turn consists of physical capital investment and human capital

investment in the immediately following period. The SSTF thus constitutes the revenue

source for government fiscal expenditures. Let G_t denote government redistributive

transfer expenditures in period t and let S_t^g denote government saving in period t. Let a_g

denote the fraction of the SSTF that is devoted to redistributive transfers. Then period t

government redistributive transfer expenditures, G_t, and government saving, S_t^g, are

given, respectively, by

$$G_t = a_g SSTF_t; \tag{2-8}$$

$$S_t^g = (1 - a_g)SSTF_t, \tag{2-9}$$

where $0 \leq a_g \leq 1$.

Let K^g_{t-1} denote government physical capital investment in period $t + 1$ and let H_{t-1} denote government human capital investment in period $t + 1$. Let a_k denote the fraction of the government saving that goes to K^g_{t-1}. Then period $t + 1$ government physical capital investment, K^g_{t-1}, and human capital investment, H_{t-1}, are given. respectively, by

$$K^g_{t-1} = a_k S^g_t; \qquad\qquad\qquad (2\text{-}10)$$

$$H_{t-1} = (1 - a_k)S^g_t, \qquad\qquad\qquad (2\text{-}11)$$

where $0 \leq a_k \leq 1$. Initial government investments are assumed to be given by $\overline{K}^g_1 = \overline{H}_1 = 0$, implying that the SS system is implemented at the beginning of period 1.

We assume that the government redistributes G_t equally among living agents in period t in a lump-sum fashion. Thus the lump-sum transfer in period t, denoted by T_t, is

$$T_t = \frac{G_t}{N_{t-1} + N_t}. \qquad\qquad\qquad (2\text{-}12)$$

Also, we assume that the government capital income in period $t + 1$ accrued from the physical investment of the $a_k(1 - a_g)$ fractional amount of the SSTF in period t goes to the generation t old SS benefits. A generation t old agent in period $t + 1$ thus receives his SS benefits from two sources: One is from the SS tax payments by the generation $t + 1$ young and the other is from government capital income in period $t + 1$. The SS benefits for the generation t old agent in period $t + 1$, denoted by $b_{t + 1}$, are thus given by

$$b_{t+1} = \frac{[1 - \delta]v_{t-1}N_{t-1} + (1 + r_{t-1})K_{t-1}^g}{N_t},$$
(2-13)

for $t \geq 1$. For the given set of initial government capital investments $\overline{K}_1^g = \overline{H}_1 = 0$. the SS benefits of each generation 0 old agent, b_1, is given by

$$b_1 = \frac{[1 - \delta]v_1N_1 + (1 + r_1)\overline{K}_1^g}{N_0} = \frac{[1 - \delta]v_1N_1}{N_0}.$$
(2-14)

As Figure 2.2 details, a particular SS system for the economy at hand is characterized by different specifications of the four parameters τ, δ, a_g, and a_k, which respectively represent the SS tax rate, the SSTF fraction, the government transfer fraction, and the government physical capital investment fraction. From now on, we refer

21
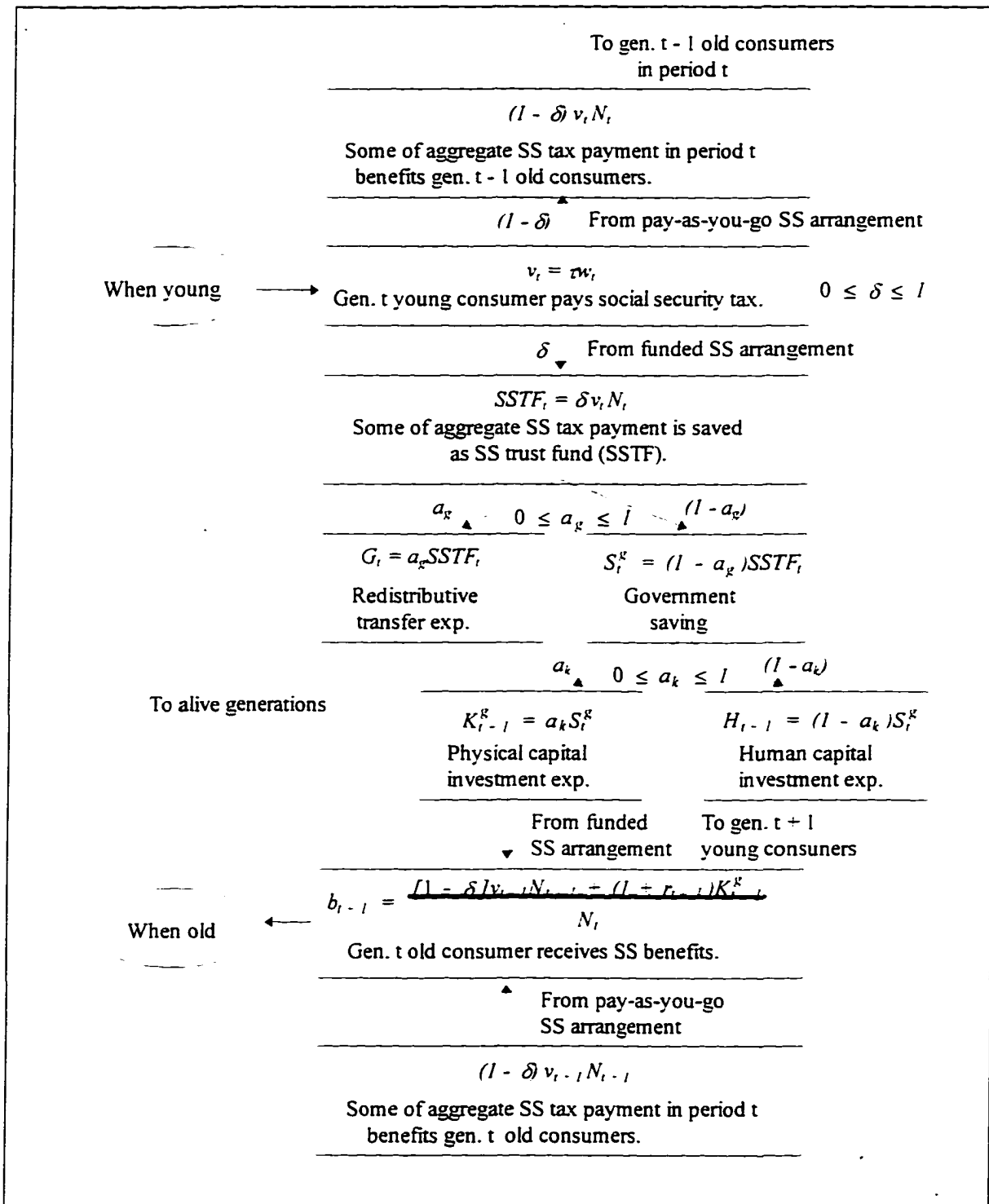
Figure 2.2: The structure of the social security (SS) system from the perspective of generation t

to (τ, δ, a_g, a_k) as the SS system. Thus, for instance, given τ, a_g, and a_k, a SS can be distinguished as an unfunded system ($\delta = 0$), a partially funded system ($0 < \delta < 1$), or a fully funded system ($\delta = 1$). Under the unfunded (pay-as-you-go) SS system, aggregate SS tax payments made by the generation t young agents in period t are simply transferred to the generation t - 1 old agents in period t.

It is often considered that a pay-as-you-go SS system, as an intergenerational transfer program, earns an *implicit* rate of return on tax payments that critically depends on the biological interest rate n. So, as demography changes in such a way that the population decreases, the implicit return on the SS tax payments decreases and *vice versa*. This popular notion is potentially misleading in the sense that only a certain set of limited market interactions are considered, as will be clarified further later on. It is also thought by many that, when the social security system is run on a fully funded basis, the rate of return on the tax payments will be exactly the same as the market interest rate in period t+1, presuming that there is only one interest rate. This latter notion is, in fact, very similar to the previous notion in the sense that it may have almost equal possibility of being misleading. It does hold under certain restricted sets of conditions. For example, it holds if all of the SS tax payments made by the generation t young in period t are saved and invested as physical capital in period t + 1, and capital income on the investment goes to the SS benefits of old agents in period t + 1.

The government fiscal policy regarding the use of the SSTF is described by different specifications of the policy parameters a_g and a_k. For instance, for given

positive values of τ and δ, the government fiscal policy in its extreme can be distinguished as a government transfer policy only ($a_g = 1$), a government physical capital investment policy only ($a_g = 0$ and $a_k = 1$), or a human capital investment item only ($a_g = a_k = 0$). It is questioned, often without much logical justification, whether the use of the SSTF in the form of a government transfer item ($a_g = 1$) would be detrimental to SS benefits, and, more importantly, if so, then by how much. Also, it is virtually unknown what potential consequences the use of the SSTF in the form of human capital investment ($a_g = a_k = 0$) would eventually lead to. Should not the use of the SSTF then be restricted in such a fashion that the SSTF is invested as physical capital and the return on this particular investment is distributed out to its presumed owners, the SS tax payers? With the question being granted, can the use of the SSTF in the form of physical capital investment ($a_g = 0$ and $a_k = 1$) justify the very existence of the SSTF itself, especially when the investment raises the relative abundance of physical capital stock to labor stock and thus depresses the other source of old age income, the return on personal saving?

2.5. Young Agent's Optimization Problem

The consumption and saving decisions of a generation t young agent are endogenously determined by his optimizing behavior. The optimizing behavior is summarized by maximizing his life-time utility within his means, taking relative prices and the SS system as given. More precisely, the generation t young agent takes as given the wage rate w_t in period t and the rate of return r_{t+1} on saving in period $t + i$, the SS tax

rate τ when young and the SS benefit b_{t-1} when old, and the lump-sum transfers T_t in period t and T_{t+1} in period t + 1. The lifetime utility maximization problem faced by a generation t young agent is

$$\text{Max } \beta \log c_t^y + (1 - \beta) \log c_{t-1}^o, \qquad (2\text{-}15)$$

with respect to c_t^y, c_{t-1}^o, and s_t^p subject to the budget and non-negativity constraints

$$c_t^y + s_t^p = [1 - \tau]w_t + T_t; \qquad (2\text{-}16)$$

$$c_{t-1}^o = (1 + r_{t-1})s_t^p + b_{t-1} + T_{t-1}; \qquad (2\text{-}17)$$

$$s_t^p \geq 0, c_t^y \geq 0, c_{t-1}^o \geq 0. \qquad (2\text{-}18)$$

Given $(w_t, r_{t+1}, \tau, b_{t+1}, T_t, T_{t+1})$, we can use the two budget constraints to express the consumption levels in the utility function in terms of the personal saving s_t^p in period t. The generation t young agent's optimization problem can then be stated as:

$$\text{Max } \beta \log([1-\tau]w_t + T_t - s_t^p) + (1-\beta)\log((1+r_{t-1})s_t^p + b_{t-1} + T_{t-1}), \quad (2\text{-}19)$$

with respect to s_t^P subject to the non-negativity constraints

$$s_t^P \geq 0; \tag{2-20}$$

$$[1 - \tau]w_t + T_t - s_t^P \geq 0; \tag{2-21}$$

$$(1 + r_{t-1})s_t^P + b_{t-1} + T_{t-1} \geq 0. \tag{2-22}$$

Assuming an interior solution, the first-order necessary condition for the problem (2-19)-(2-22) is

$$\frac{\beta}{([1 - \tau]w_t + T_t - s_t^P)} = \frac{(1 - \beta)(1 + r_{t-1})}{(1 + r_{t-1})s_t^P + b_{t-1} + T_{t-1}}, \tag{2-23}$$

by which his optimal saving decision is summarized. That is, rearranging (2-23), the generation t young agent's optimal saving is given by

$$\begin{aligned} s_t^P &= (1 - \beta)([1 - \tau]w_t + T_t) - \frac{\beta b_{t-1} + T_{t-1}}{(1 + r_{t-1})} \\ &= s^P(w_t, r_{t+1}, \tau, b_{t-1}, T_t, T_{t-1}). \end{aligned} \tag{2-24}$$

Conversely, if the right side of (2-24) can be shown to be strictly positive and less than $[1 - \tau]w_t + T_t$, then an interior solution exists. [8]

The demand function for the generation t young agent's saving (2-24), together with (2-14)-(2-16), leads us to his optimal demands for consumption levels in both periods and his optimized utility as functions of w_t, r_{t-1}, τ, b_{t-1}, T_t, and T_{t-1}:

$$c_t^y = [1 - \tau]w_t + T_t - s_t^p(w_t, r_{t-1}, \tau, b_{t-1}, T_t, T_{t-1})$$
$$= c_t^y(w_t, r_{t-1}, \tau, b_{t-1}, T_t, T_{t-1}); \tag{2-25}$$

$$c_{t-1}^o = (1 + r_{t-1})s_t^p(w_t, r_{t-1}, \tau, b_{t-1}, T_t, T_{t-1}) + b_{t-1} + T_{t-1}$$
$$= c_{t-1}^o(w_t, r_{t-1}, \tau, b_{t-1}, T_t, T_{t-1}); \tag{2-26}$$

$$U_t = \beta \log c_t^y + (1 - \beta)\log c_{t-1}^o(1 + r_{t-1})$$
$$= U(w_t, r_{t-1}, \tau, b_{t-1}, T_t, T_{t-1}). \tag{2-27}$$

We assume that the consumption of the generation 0 old agent in period 1 is given by

$$c_1^o = (1 + r_1)\frac{\overline{K_1^p}}{\overline{N_0}} + b_1 + T_1. \tag{2-28}$$

---

[8] It is assumed through-out the rest of the theoretical potion of this dissertation that exogenous variables are set in such a way that these restriction hold. In all simulation experiments, the exogenous variables will be specifically set to ensure these restrictions hold.

Notice from (2-28) that the introduction of a SS system in period 1 may provide the generation 0 old agents a potential wealth windfall. That is, as long as $a_g > 0$, the sum of the SS benefit and the lump-sum transfer for the generation 0 old consumers is greater than zero. When the SS system is run on a fully funded basis ($\delta = 1$) and the SSTF is invested as either physical or human capital, the generation 1 old are indifferent to whether or not the SS system exists in the model economy.

2.6. Production Technology

In each period t the economy produces a time-dated good, using physical capital and labor input in a production technology which is assumed to take the Cobb-Douglas production form

$$Y_t = F(K_t, L_t) = A K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{2-29}$$

Here Y_t measures gross national output (GNP) in period t; A measures total factor productivity; K_t denotes the aggregate physical capital stock of the economy in period t, which is assumed to depreciate completely in each period; L_t denotes the aggregate effective labor supply of the economy in period t. The production function exhibits constant-returns-to-scale technology and satisfies the following "Inada conditions": The function is continuous, strictly increasing, strictly concave over nonnegative K and L, and

twice continuously differentiable over positive K and L. Also, $F(K, L) = 0$ if either K or L is zero, and, for L (K) > 0, F_K (F_L) approaches infinity as K (L) approaches zero and approaches zero as K (L) approaches infinity.

The aggregate physical capital stock K_t in period t consists of the private physical capital stock K_t^p in period t and the government physical capital stock K_t^g in period t:

$$K_t = K_t^p + K_t^g, \tag{2-30}$$

where the aggregate physical capital stock K_1 in the initial period 1 is given by

$$K_1 = \overline{K}_1^p + \overline{K}_1^g = \overline{K}_1^p. \tag{2-31}$$

The aggregate effective labor supply of the economy in period t is given by

$$L_t = e_t N_t, \tag{2-32}$$

where N_t is the raw labor force in period t which is, in fact, the population of generation t young, and e_t measures the efficiency per unit of the raw labor force in period t due to government human capital investment in the same period. The effective labor stock in period 0 is assumed to be equal to the raw labor force in period 0.

The technology of human capital in period t is postulated in such a way that labor efficiency in period t is a function of the government's human capital investment in period t and the effective labor force in period t - 1. More precisely, the efficiency in period t is related positively to the level of human capital investment in period t and negatively to the effective labor force in period t - 1. The specification of the technology is given by

$$e_t = \exp^{\lambda(H_t, L_{t-1})},$$
(2-33)

where λ denotes an efficiency parameter. When the value of λ increases, human capital becomes more productive. Also, by assumption, $\overline{H}_1 = 0$, which implies that $e_1 = 1$. Note that, for the same reason, in the complete absence of government human capital investment, labor efficiency is always equal to one.

The profit maximizing behavior of competitive firms is assumed to determine the rate of return on physical capital and the effective wage rate in each period t. We assume that the labor and physical capital markets continuously clear. That is, labor demanded by firms is equal to the effective labor supply and capital is always fully utilized. Consequently, physical capital and effective labor are paid in accordance with their marginal productivities as follows:

$$(1 + r_t) = F_K(K_t, L_t) = \alpha A(K_t/L_t)^{\alpha-1};$$
(2-34)

$$\tilde{w}_t = F_L(K_t, L_t) = (1 - \alpha) A(K_t/L_t)^\alpha, \qquad (2\text{-}35)$$

where $\tilde{w}_t$ is the effective wage rate per unit of effective labor in period t.

Finally the relationship between the effective wage rate, $\tilde{w}_t$, and the wage rate actually paid to each generation t young agent, w_t, is given by

$$w_t = e_t \tilde{w}_t. \qquad (2\text{-}36)$$

2.7. Capital Accumulation

In the product market the demand for good in each period t needs to be equal to the supply in each period for all $t \geq 1$. Equivalently, total private saving in period t plus the a_k fraction of government saving in period t must equal the physical capital stock in period $t + 1$. Note that government saving in period t consists of government physical and human capital investments in period $t + 1$. The private and government physical capital accumulation processes are thus given by

$$K_{t+1}^p = N_t s_t^p; \qquad (2\text{-}37)$$

$$K_{t+1}^g = a_k S_t^g. \qquad (2\text{-}38)$$

In (2-37) the left hand side is the private physical capital stock in period $t + 1$ and the right hand side is private saving, the sum of all personal saving in period t. In (2-38) the left hand side is the government physical capital stock in period $t + 1$ and the right hand side is the a_k fraction of government saving in period t.

The government human capital accumulation process is given by

$$H_{t+1} = (1 - a_k)S_t^g, \qquad (2-39)$$

where the human capital investment in period $t + 1$ on the left side is equal to the $(1 - a_k)$ fraction of government saving in period t on the right side.

2.8. Definition of Equilibrium

An equilibrium for the economic model will now be defined. Attention will be restricted to equilibria for which the optimal consumption and savings levels of each generation t young agent are strictly positive. Given $(\overline{K}_1^p, \overline{K}_1^g = \overline{H}_1 = 0, \overline{N}_0, n)$ and a SS system (τ, δ, a_g, a_k) satisfying $0 \le \tau, \delta, a_g, a_k \le 1$, an equilibrium is a collection of sequences $\left\{c_t^y, c_t^o, s_t^p\right\}_{t=1}^{t=\infty}, \left\{v_t, SSTF_t, G_t, T_t, S_t^g, b_t\right\}_{t=1}^{t=\infty}$, and $\left\{K_t^p, K_t^g, K_t, H_t, N_t, L_t, r_t, \tilde{w}_t, w_t\right\}_{t=1}^{t=\infty}$ such that the following conditions hold:

1. The capital and labor sequences $\left\{K_t, L_t\right\}_{t=1}^{t=\infty}$ satisfy

$$(1 + r_t) = \alpha A(K_t/L_t)^{\alpha-1}, \qquad \forall t \geq 1;$$

$$\widetilde{w}_t = (1 - \alpha) A(K_t/L_t)^{\alpha}, \qquad \forall t \geq 1,$$

where

$$w_t = e_t \widetilde{w}_t, \qquad \forall t \geq 1;$$

$$e_t = \exp^{\lambda H_t L_{t-1}}, \qquad \forall t \geq 1.$$

2. The consumption and saving sequences $\left\{c_t^y, c_t^o, s_t^p\right\}_{t=1}^{t=\infty}$ solve the lifetime utility

maximization problem faced by each generation t young agent by satisfying

$$s_t^p = s^p(w_t, r_{t-1}, \tau, b_{t-1}, T_t, T_{t-1}) > 0, \qquad \forall t \geq 1;$$

$$c_t^y = [1 - \tau]w_t + T_t - s_t^p > 0, \qquad \forall t \geq 1;$$

$$c_{t+1}^o = (1 + r_{t-1})s_t^p + b_{t-1} + T_{t-1}, \qquad \forall t \geq 1,$$

and the consumption of each generation 0 old agent in period 1 is given by

$$c_1^o = (1 + r_1)\frac{\overline{K}_1^p}{\overline{N}_0} + b_1 + T_1.$$

3. Physical and human capital are accumulated by satisfying

$$K_{t-1}^p = N_t s_t^p, \qquad\qquad \forall t \geq 1;$$

$$K_{t-1}^g = a_k S_t^g, \qquad \forall t \geq 1;$$

$$K_{t-1} = K_{t-1}^p + K_{t-1}^g, \qquad\qquad \forall t \geq 1,$$

where the aggregate physical capital stock in the initial period 1 is given by

$$K_1 = \overline{K}_1^p + \overline{K}_1^g = \overline{K}_1^p.$$

4. The SS system satisfies the following conditions:

$$v_t = \tau w_t, \qquad\qquad \forall t \geq 1;$$

$$SSTF_t = \delta v_t N_t, \qquad \forall t \geq 1;$$

$$G_t = a_g SSTF_t, \qquad \forall t \geq 1;$$

$$S_t^g = (1 - a_g)SSTF_t, \qquad \forall t \geq 1;$$

$$T_t = \frac{G_t}{N_{t-1} + N_t}, \qquad \forall t \geq 1;$$

$$b_{t-1} = \frac{[1 - \delta]v_{t-1}N_{t-1} + (1 + r_{t-1})K_{t-1}^g}{N_t}, \qquad \forall t \geq 1.$$

and the SS benefits of each generation 0 old agent, b_1, are given by

$$b_1 = \frac{[1 - \delta]v_1 N_1 + (1 + r_1)\overline{K}_1^g}{\overline{N}_0} = \frac{[1 - \delta]v_1 N_1}{\overline{N}_0}.$$

2.9. Equilibrium in a Reduced Per-Capita Form

Using the above micro-based macroeconomic model, our basic purpose is to

characterize general equilibrium and intergenerational equity effects of alternative SS

systems (τ, δ, a_g, a_k). Intergenerational equity consequences can be investigated from various viewpoints depending on whether the focus is put on actuarial status within a SS system or the relative welfare of generations as measured by lifetime utility. To investigate these matters, we first need to investigate the dynamic behavior of physical and human capital for the economic model at hand.

The definition of the physical capital stock (2-30), together with the private and government physical capital accumulation processes (2-37) and (2-38), implies that

$$K_{t-1} = N_t s_t^p + a_k (1 - a_g) S_t^g, \tag{2-40}$$

where the initial physical capital stock K_1 is given by

$$K_1 = \overline{K}_1^p + \overline{K}_1^g = \overline{K}_1^p + 0 = \overline{K}_1^p. \tag{2-41}$$

Recalling our assumption that capital depreciates completely at the end of each period, the left hand side of (2-40) is net investment in aggregate physical capital in period $t + 1$. The first term on the right hand side is the private saving of the generation t young agents in period t and the second is the fraction of government saving S_t^g in period t that results in physical capital in period $t + 1$.

The physical capital accumulation equation (2-40), the form (2-24) for optimal saving, the SS auxiliary conditions (2-7), (2-9), and (2-10), imply that

$$K_{t-1} = N_t \left(w_t + T_t - [1 - a_k(1 - a_g)\delta]\tau w_t - \beta \frac{b_{t-1} + T_{t-1}}{1 + r_{t-1}} \right)$$

$$= K(w_t, r_{t-1}, sss, b_{t-1}, T_t, T_{t-1}),$$

(2-42)

where $sss = (\tau, \delta, a_g, a_k)$ denotes the SS system.

Furthermore, eliminating b_{t-1}, T_t, and T_{t-1} in (2-42) using the SS auxiliary

conditions (2-6)-(2-10) and (2-12)-(2-14), equation (2-42) can further be reduced to

$$K_{t-1} = N_t(1 - \beta)\left(1 - \tau\left(1 - (\frac{n+1}{n+2}a_g + a_k(1 - a_g))\delta\right)\right)w_t$$

$$- (n+1)N_t\left(\beta\tau\left(1 - (1 - \frac{1}{n+2}a_g)\delta\right)\right)\frac{w_{t-1}}{1 + r_{t-1}}$$

(2-43)

$$= K(w_t, w_{t-1}, r_{t-1}, sss).$$

Finally, using the firm profit maximization conditions (2-34) and (2-35), together

with the labor force relationship (2-32), the wage relationship (2-33), and the last SS

auxiliary condition (2-11), the physical capital accumulation equation (2-43) can be

expressed as follows:

$$K_{t-1} = N_t \frac{A(1-\alpha)(1-\beta)\left(1 - \tau\left(1 - (\frac{n-1}{n+2}a_g + a_k(1-a_g))\delta\right)\right)}{1 + \frac{1-\alpha}{\alpha}\beta\tau\left(1 - (1 - \frac{1}{n+2}a_g)\delta\right)} e_t^{1-\alpha}[K_t^\alpha \cdot N_t^{-\alpha}]$$

$$= K(K_t, N_t, e_t, sss). \tag{2-44}$$

The physical capital stock in period t + 1 is a function of physical capital stock in period t, raw labor in period t, and labor efficiency in period t, which in turn depends on physical capital stock in period t - 1, raw labor in period t - 1, and labor efficiency in period t - 1. As the efficiency term in each period t is successively substituted out, the physical capital stock in period t + 1 is expressed as a function of physical capital and raw labor in all previous periods, in addition to labor efficiency e_1 in period 1.[9] Finally through a recursive substitution out of physical capital and raw labor over time using (2-30) and (2-1), the physical capital stock in period t + 1 can ultimately be expressed as a function only of exogenously given parameters and initial conditions. Note that, when $a_k=1$, implying that there is no government human capital investment over time, the physical capital accumulation equation (2-44) reduces to a simple form in which the physical capital stock in period t + 1 is a function only of physical capital stock and raw labor in period t, in addition to the SS system. That is,

---

[9] Recall that $H_1 = 0$ and $L_0 = \overline{N}_0 > 0$, by assumption. Thus, $e_1 = \exp(\lambda H_1/L_0) = 1$.

$$K_{t-1} = K(K_t, N_t, sss).$$ (2-45)

Let k_t denote per capita physical capital in period t, i.e., the ratio (K_t/L_t) of physical capital to effective labor in period t. Let y_t denote per capital GNP in period t. Dividing both sides of (2-44) by L_{t-1} then gives

$$k_{t-1} = K_{t-1}/L_{t-1}$$

$$= \frac{A(1 - \alpha)(1 - \beta)\left(1 - \tau\left(1 - (\frac{n+1}{n+2}a_g + a_k(1 - a_g))\delta\right)\right)}{(n+1)\left(1 + \frac{1-\alpha}{\alpha}\beta\tau\left(1 - (1 - \frac{1}{n+2}a_g)\delta\right)\right)}\left[\frac{e_t}{e_{t-1}}\right]k_t^\alpha$$ (2-46)

$$= k_{t-1}(k_t, e_t / e_{t-1}, sss).$$

Using (2-33) and various other previously given model relations to substitute out the labor efficiency terms in (2-46), the per capita physical capital in period t + 1 can be expressed as a function of per capita physical capital in period t and period t - 1. Finally through the recursive substitution out of past per capita physical capital stocks, per capita physical capital stock in period t + 1 can be expressed as a function of the initial per capita physical capital in period 1, k_1, along with exogenously given initial conditions and parameters, where

$$k_1 = \frac{\overline{K}_1^p + \overline{K}_1^g}{e_1(\widehat{H}_1 / L_0)(n + 1)\overline{N}_0} = \frac{\overline{K}_1^p}{(n + 1)\overline{N}_0}. \qquad (2\text{-}47)$$

In summary, the evolution of the economy at hand is fully accounted for by the per capita version (2-46) of the physical capital accumulation equation (2-44). Specifically, in each period $t \geq 1$,

$$k_{t-1} = \frac{C}{D} \cdot \frac{e_t}{e_{t-1}} \cdot k_t^\alpha, \qquad (2\text{-}48)$$

where

$$e_t = e_t(k_{t-1}) = \exp^{\lambda(1 - a_k)(1 - a_g)\tau\delta(1 - \alpha)k_{t-1}^\alpha}, \ t \geq 2;$$

$$e_i = 1;$$

$$C = A(1 - \alpha)(1 - \beta)\left(1 - \tau\left(1 - (\frac{n + 1}{n + 2}a_g + a_k(1 - a_g))\delta\right)\right); \qquad (2\text{-}49)$$

$$D = (n + 1)\left(1 + \frac{1 - \alpha}{\alpha}\beta\tau\left(1 - (1 - \frac{1}{n + 2}a_g)\delta\right)\right). \qquad (2\text{-}50)$$

The classification of variables for this state equation is as follows:

Time-t endogenous variable ($t \geq 1$): k_{t+1};

Time-t predetermined variables ($t \geq 3$): k_t, k_{t-1};

Exogenous variables: $\overline{k}_1 = \dfrac{\overline{K}_1^p}{(1+n)\widetilde{N}_0}$, n, A, α, β, τ, δ, a_t, a_k;

Admissible Conditions: $0 \le \tau$, δ, a_g, $a_k \le 1$, $0 < \alpha$, $\beta < 1$, $0 < \overline{K}_1^p$, $0 < \overline{N}_0$, $-1 < n$.

Solving (2-48) in each period $t \ge 1$, the sequence of the equilibrium per capita

physical capital stocks $\{k\}_{t=1}^{t=\infty}$ is obtained. Once per capita physical capital stocks are

determined over time, the equilibrium values for the sequences $\left\{c_t^y, c_t^o, s_t^p\right\}_{t=1}^{t=\infty}$,

$\left\{v_t, SSTF_t, G_t, T_t, S_t^g, b_t\right\}_{t=1}^{t=\infty}$, and $\left\{K_t^p, K_t^g, K_t, H_t, N_t, L_t, r_t, \widetilde{w}_t, w_t\right\}_{t=1}^{t=\infty}$ can be

derived using the basic model relationships together with the defining conditions for an

equilibrium.

2.10. Mathematical Verification of the Economic Model

2.10.1. Does the product market clear in each period t? Given private agent

and firm optimization, and initial aggregate physical and human capital stocks, and the

definitions of aggregate physical and human capital stocks, the accumulation process

assumed for capital already implies that the product market clears in each period $t \ge 1$. It

is, however, worth while to see this through mathematical expression for, by doing so, we

may gain additional understanding about the sequence of equilibrium allocations over

time. Let C_t denote aggregate consumption in period t and let S_t denote aggregate saving

in period t. It then suffices to show that

$$C_t + S_t = Y_t, \qquad t \geq 1, \qquad (2\text{-}51)$$

where product Y_t is given by (2-29).

For any period $t \geq 2$, aggregate consumption and saving are given by

$$
\begin{aligned}
C_t &= c_t^y N_t + c_t^o N_{t-1} \\
&= ([1 - \tau]w_t + T_t - s_t^p)N_t + ((1 + r_t)s_{t-1}^p + T_t + b_t)N_{t-1} \\
&= ([1 - \tau]w_t - s_t^p)N_t + ((1 + r_t)s_{t-1}^p + b_t)N_{t-1} + G_t \\
&= (w_t - s_t^p)N_t + (1 + r_t)(K_t^p + K_t^g) - SSTF_t + a_g SSTF_t \\
&= (w_t - s_t^p)N_t + (1 + r_t)K_t - (1 - a_g)SSTF_t; \qquad (2\text{-}52)
\end{aligned}
$$

$$
\begin{aligned}
S_t &= s_t^p N_t + S_t^g \\
&= s_t^p N_t + (1 - a_g)SSTF_t. \qquad (2\text{-}53)
\end{aligned}
$$

Thus, the sum of aggregate consumption and saving in period t is expressed as follows:

$$
\begin{aligned}
C_t + S_t &= [(w_t - s_t^p)N_t + (1 + r_t)K_t - (1 - a_g)SSTF_t] + \\
&\quad [s_t^p N_t + (1 - a_g)SSTF_t] \\
&= w_t N_t + (1 + r_t)K_t \\
&= e_t \tilde{w}_t N_t + (1 + r_t)K_t
\end{aligned}
$$

$$= ((1 - \alpha)AK_t^\alpha L_t^{-\alpha})L_t + (\alpha AK_t^{\alpha-2}L_t^{1-\alpha})K_t$$

$$= AK_t^\alpha L_t^{1-\alpha}$$

$$= Y_t . \tag{2-54}$$

Finally, for period 1, aggregate consumption and saving are given by

$$C_1 = c_1^y N_1 + c_1^o N_0$$

$$= ([1 - \tau]w_1 + T_1 - s_1^p)N_1 + ((1 + r_1)\overline{K}_1^p + T_1 + b_1)N_0$$

$$= ([1 - \tau]w_1 - s_1^p)N_1 + ((1 + r_1)s_{t-1}^p + b_1)N_0 + G_1$$

$$= (w_1 - s_1^p)N_1 + (1 + r_1)(\overline{K}_1^p + \overline{K}_1^g) - SSTF_1 + a_g SSTF_1$$

$$= (w_1 - s_1^p)N_1 + (1 + r_1)K_1 - (1 - a_g)SSTF_1; \tag{2-55}$$

$$S_1 = s_1^p N_1 + S_1^g$$

$$= s_1^p N_1 + (1 - a_g)SSTF_1. \tag{2-56}$$

Thus, the sum of aggregate consumption and saving in period 1 is

$$C_1 + S_1 = [(w_1 - s_1^p)N_1 + (1 + r_1)K_1 - (1 - a_g)SSTF_1] +$$

$$[s_1^p N_1 + (1 - a_g)SSTF_1]$$

$$= w_1 N_1 + (1 + r_1)K_1$$

$$= e_1 \tilde{w}_1 N_1 + (1 + r_1) K_1$$

$$= \tilde{w}_1 L_1 + (1 + r_1) K_1$$

$$= ((1 - \alpha) A K_1^{\alpha} L_1^{-\alpha}) L_1 + (\alpha A K_1^{\alpha-2} L_1^{1-\alpha}) K_1$$

$$= A K_1^{\alpha} L_1^{1-\alpha}$$

$$= Y_1 . \tag{2-57}$$

It follows from (2-54) and (2-57) that the product market clearing condition is satisfied in each period.

2.10.2. Is the SS system self-financing? For a given SS system (τ, δ, a_g, a_k), we define several measures of the SS system in order to establish a government budget constraint. Aggregate SS tax payments in period t are the sum of individual SS tax payments in period t. Aggregate SS tax payments in period t, denoted by SST_t, are thus given by

$$SST_t = \tau w_t N_t . \tag{2-58}$$

SS revenues in period t are the sum of aggregate SS tax payments in period t and capital income in period t accrued from government physical capital investment. SS revenues in period t, denoted by SSR_t, are thus given by

$$SSR_t = SST_t + (1 + r_t)K_t^g. \tag{2-59}$$

Aggregate SS benefits in period t are the sum of individual SS benefits in period t. Aggregate SS benefits in period t, denoted by SSB_t, are thus given by

$$SSB_t = b_t N_{t-1}. \tag{2-60}$$

SS expenditures in period t are the sum of aggregate SS benefits in period t, government redistributive transfer expenditures G_t in period t, and government saving S_t^g in period t. SS expenditures in period t, denoted by SSE_t, is then given by

$$SSE_t = SSB_t + G_t + S_t^g. \tag{2-61}$$

The government's budget constraint in each period t requires that aggregate SS revenues SSR_t be equal to aggregate SS expenditures SSE_t, which can be expressed by the simple equation[10]

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[10] Kotlifoff's government intertemporal budget constraint in his *generational accounting* can be interpreted as "sum of present value of SSR over time = sum of present value of SSE over time."
On the other hand, the budget constraint of the U.S. SS system is expressed as

$$SSR_t + \frac{SSR_{t+1}}{1 + r_{t+1}} = SSE_t + \frac{SSE_{t+1}}{1 + r_{t+1}},$$

where the length of a period is 75 years and the government use regarding the SSTF is not specified except for purchases of U.S. Treasury Securities. See Kotlikoff (1992) and "The annual report of U.S. SS system (1996) for respective budget constraint in details.

$$SSR_t = SSE_t, \qquad t \geq 1. \qquad\qquad (2\text{-}62)$$

Using the SS auxiliary conditions (2-6)-(2-14), we can show the following equalities: For periods $t \geq 2$,

$$
\begin{aligned}
SSE_t &= SSB_t + G_t + S_t \\
&= b_t N_{t-1} + SSTF_t \\
&= [(1 - \delta)\tau]w_t N_t + (1 + r_t)K_t^g + SSTF_t \\
&= [(1 - \delta)\tau]w_t N_t + (1 + r_t)K_t^g + \delta\tau w_t N_t \\
&= \delta\tau w_t N_t + (1 + r_t)K_t^g \\
&= SST_t + (1 + r_t)K_t^g \\
&= SSR_t.
\end{aligned}
\qquad\qquad (2\text{-}63)
$$

For period $t = 1$,

$$
\begin{aligned}
SSE_1 &= SSB_1 + G_1 + S_1 \\
&= b_1 \overline{N}_0 + SSTF_1 \\
&= [(1 - \delta)\tau]w_1 N_1 + SSTF_1 \\
&= [(1 - \delta)\tau]w_1 N_1 + \delta\tau w_1 N_1
\end{aligned}
$$

$$= \delta \tau w_t N_t$$

$$= SST_t$$

$$= SSR_t. \tag{2-64}$$

It follows from (2-62) and (2-64) that the government budget constraint is satisfied over time, which suffices to show that the SS system is self-financing.

2.11. Nature of Equilibrium

In Diamond Economy, the existence of a unique stable steady state equilibrium is ensured by imposing various regularity conditions, which can be summarized as follows: First, for (Walrasian) stability in the capital market the demand curve for (physical) capital is assumed to be steeper than the supply curve, implying that, given a positive (physical) capital stock in any period t, an increase in the wage rate, w_t, in period t increases personal saving, s_t^p, in period t and also an increase in the rate of return on (physical) capital r_{t+1} in period $t + 1$ increases s_t^p. See Diamond (1965, right side of Diagram 2, p. 1133). Then, Diamond (1965, Diagram 3, p. 1133) adds another assumption $0 < dr_{t+1} / dr_t < 1$.

Can we apply this assumption to the present economic model to ensure the same result? Unfortunately not. It is because the history of our economic model can not be traced down as done in the Diamond Economy. That is, in the present economic model, the saving function (2-24) in each period t is not expressed in terms of only the wage rate

in period t and the rate of return on physical capital in period t + 1. However, the unique

steady state equilibrium for the economic model at hand can explicitly determined in the

following way. First, solving the dynamic state equation (2-48) with $k_{t+1} = k_t$, the steady

state per-capita physical capital stock $\bar{k}$ is

$$\bar{k} = \left(\frac{C}{D}\right)^{\frac{1}{1-\alpha}}, \tag{2-65}$$

where C is given by (2-49) and D is given by (2-50).

[Note that, for given parameters and a given SS system (τ, δ, a_g, a_k), $\bar{k}$ is uniquely

determined.]

Second, if the derivative (dk_{t-1} / dk_t) is less than one in absolute value in a

neighborhood of the unique steady state equilibrium $\bar{k}$, then $\bar{k}$ is locally stable, meaning

that all equilibrium paths that start with an initial per capita physical capital level k_1 close

to $\bar{k}$ must eventually converge to $\bar{k}$. Once a unique steady state per capita physical

capital stock $\bar{k}$ is ensured for a given set of initial conditions and basic economic

parameters, its local stability property is easily proved by using a local linear

approximation technique for the dynamic state equation (2-48).[11]

---

[11] A rough sketch of its proof is as follows: Given admissible exogenously given specifications (p. 40) let
$\bar{k} > 0$ denote the unique stationary solution in the admissible solution set, denoted by V, conditional on the
admissible solution specifications. Define a function Z: $R_- \to R_-$ by $Z(k) \equiv \frac{C}{D}k^\alpha \equiv \frac{C}{D}f(k)$ where

Finally, for the unique stable steady state per-capita physical capital, The steady

state equilibrium values $\{\bar{c}^y, \bar{c}^o, \bar{s}^p\}$ and $\{\bar{r}, \widetilde{\bar{w}}, \bar{e}, \bar{w}\}$ can then be derived via the model

relations and equilibrium conditions. For instance, using (2-34), the steady state

equilibrium rate of return on physical capital is given by

$$\bar{r} = \alpha A \bar{k}^{\alpha - 1} - 1 = \alpha A \left(\frac{D}{C}\right)^{\frac{\alpha - 1}{\alpha - 1}} - 1 = \alpha A \left(\frac{D}{C}\right) - 1, \tag{2-66}$$

2.12. Features of the Economic Model

Feature 1: Any steady state equilibrium satisfying $\bar{r} = n$ for the economic model at hand

is, by definition, a golden rule equilibrium.[12] It can be shown that $\bar{r} = n$ if and

only if

$$\tau(1 - M\delta) = \frac{(1 - \alpha)(1 - \beta) - \alpha}{1 - \alpha}, \tag{2-67}$$

$Z(\bar{k}) = \bar{k}$; $Z'(k) = \frac{C}{D} f'(k)$ with $0 < Z'(\bar{k}) < 1$. Using Taylor's Theorem, expand $Z(k)$ about $\bar{k}$ to obtain the linear approximation system (LAS) for the BCDE as follows:

$$k_{t+1} \cong Z'(\bar{k})[k - \bar{k}] \text{ for } k \approx \bar{k}.$$

Here, since $Z'(\bar{k})$ is a positive real number less than 1, 0 is a stable stationary solution for this LAS. Noting that 0 is a stable stationary solution for the LAS implies $\bar{k}$ is a locally stable solution for the BCDE relative to the admissible solution set V, this suffices to show that $\bar{k}$ is locally stable.

[12] Samuelson (1958) and Diamond (1965) show that, in pure exchange and production overlapping-generations model, respectively, decentralized market allocation may be dynamically inefficient, which is originally proved by Phelps (1961).

where

$$M = (1 - \beta)\left[\frac{n + 1}{n + 2}a_g + (1 - a_g)a_k\right] + \beta\left[1 - \frac{1}{n + 2}a_g\right]. \qquad (2\text{-}68)$$

In particular, in the absence of a SS system ($\tau = 0$), $\bar{r} = n$ if and only if

$$\frac{\alpha}{1 - \alpha} = 1 - \beta. \qquad (2\text{-}69)$$

Feature 2: As long as the SSTF is invested under a "physical investment only" fiscal

policy ($a_g = 0$ and $a_k = 1$), private saving S^p and the SSTF in period t have the

same effect on capital accumulation over time. Also, as long as government

saving S^g is invested as physical capital K ($a_k = 1$), private saving S^p and

government saving S^g in period t have the same effect on physical capital

accumulation over time.

Feature 2 is obvious because agents are indifferent regarding who does the

saving[13] as long as the saving yields to the same return to them. Since a SS arrangement

(τ, δ) is non-distortionary in this case, Feature 2 explains why, when $a_g = 0$ and $a_k = 1$, the

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[13] See Blanchard and Fischer (1989, p. 111).

introduction of a fully funded SS system ($\delta = 1$) has no effect on aggregate saving and capital accumulation over time.

Feature 3: As long as the SSTF is invested under a "physical investment only" fiscal policy ($a_g = 0$ and $a_k = 1$), a SS system is distinguished only by its effective SS tax rate $\tilde{\tau} = \tau(1 - \delta)$. The separate values of τ and δ do not matter as long as $\tilde{\tau} = \tau(1 - \delta)$ remains unchanged.

Clearly, Features 1 and 2 suffice to show why Feature 3 holds. One particular implication of Feature 3 is that, roughly put, a SS system with high τ and high δ could be equivalent to a SS system with low τ and low δ in the sense that the same dynamic path for the per capita physical capital stock results. The choice of the individual SS parameters τ and δ matters only when it changes the value of $\tilde{\tau} = \tau(1 - \delta)$.

CHAPTER 3. SIMULATIONS

3.1. Key Questions

The current chapter focuses on the general equilibrium effects of alternative social security (SS) systems for the economic model developed in Chapter 2, particularly concerning SS funding policy and its resulting consequences for intergenerational equity. Since the economic model is too complex to allow a detailed analytical characterization of the economy's responses to changes in SS systems, we compute the dynamic equilibrium allocation to determine how different SS systems influence the time paths of, for instance, the physical capital/(effective) labor ratio, GNP, and measures of intergenerational equity.

The simulation approach can, however, pose a potential danger in the sense that a simulation result, based on a particular initialization and parameterization of a model, may be misleading. More precisely, a simulation study could focus on a certain range of parameters to justify a particular point of view. In order to minimize this danger, we first raise a series of key questions that will direct the course of the simulation experiments to be undertaken.

Question 1: *How will the economy respond to different SS arrangements, from unfunded*

to funded, as characterized by the SS tax rate τ and the social security trust

fund (SSTF) fraction δ?

In a famous debate between Barro (1974) and Feldstein (1974, 1976) on the desirability of a SSTF, Barro indicates that the accumulation of the SSTF would be offset by a decrease of private saving resulting from rational consumers' super-altruistic behavior. The result is that the existence of the SSTF is *irrelevant* to his model economy.[14] Feldstein argues that an increase in national saving would occur since the incremental SSTF is not fully offset by decreases in private saving. Supporting Feldstein's argument, Aaron, Bosworth, and Burtless (1989) show, in their simulation study on the U.S. SS system, that a SSTF has a positive effect on the capital stock, raising net national product and general consumption levels.

It is difficult to evaluate these arguments because each argument is based on a different set of assumptions, especially regarding the degree of rationality of economic agents. Still, this debate clearly provides an example of the difficulty in tracking down the economic effects of the SSTF.

Question 2: *How does government fiscal policy regarding the use of the SSTF affect the economic model?*

It is often understood that, through the accumulation of the SSTF, some portion of the SS taxes contributed by an agent when young is put aside for his own SS benefits when old, blocking a shift of resources across generations so that a closer linkage

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[14] For a detailed account of Barro's argument regarding SS system, see Visaggio (1991) and Tabellini (1991).

between SS taxes and benefits is ensured. This is not necessarily the case for our economic model. Rather, we postulate that the SSTF can be used in various ways, depending on the government fiscal policy. Specifically, the SSTF can be allocated among redistributive transfers, physical capital investment, and human capital investment, and we characterize this allocation parametrically in a way that permits all possible allocations to be considered. Thus, by construction, the economic impact of the SSTF, in whatever amount, depends on what the government does with the SSTF.

For the U.S. SSTF, this matter was set forth, for example, by Schultze (1990). Schultze writes, "...*the mere accumulation of financial assets in social security trust funds does not mean that one generation is financing its own retirement and relieving the next of any burden.*"[15] Our concern is to analyze the economic and intergenerational equity consequences of alternative SS systems with government behavior regarding the use of the SSTF incorporated into the formal analysis.

Although there is no particular reason why the specification of government fiscal policy should be limited, we pay particular attention to the following special cases:

Case I: Government physical capital investment only policy ($a_g = 0$, $a_k = 1.0$);

Case II: Government redistributive transfer only policy ($a_g = 1.0$);

Case III: Government human capital investment only policy ($a_g = a_k = 0$);

Case IV: Policy-mix: ($a_g = .5$, $a_k = 1.0$).

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[15] See Schultze (1990, p. 10).

Question 3: *How useful is the SS actuarial status of an individual agent as a measure of intergenerational equity? If it is not useful, what alternative measure might be employed?*

SS actuarial status, defined as the ratio of the present value of SS benefits received to taxes paid, explains whether or not a SS arrangement is actuarially favorable for a particular generation. Note that the term is simply another way of expressing net social security wealth (NSSW)[16], defined as the net present value of the SS benefits received minus the SS taxes paid. Also, when SS taxes are replaced by wage income, the term becomes the usual replacement rate, the ratio of SS benefits received to wage income.[17]

Although SS actuarial status is one standard measure used in the analysis of SS systems, there is a question often left out---whether or not SS actuarial status is an effective measure of intergenerational equity. We provide an alternative measure from a lifetime utility perspective, referred to as relative welfare benefit (RWB), by which the comparison of intergenerational equity is possible *over* varying SS arrangements *across* periods.

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[16] See Feldstein (1974).

[17] For instance, Musgrave (1981) introduces the "fixed relative position, a fixed ratio between per-capita income (wage income) and the per-capita benefits of the old (b_{t+1}/w_t). Also, Miguel-Angel and Lopez-Garcia defines the "net replacement rate", a ratio between net wage income when young and SS benefits when young.

Question 4: *What are the consequences of alternative SS arrangements and government uses of the SSTF for intergenerational equity measured in terms of the time profiles of the SS actuarial status and RWB? Does a build-up of the SSTF necessarily improve intergenerational equity in terms of either these two measures?*

In recent studies of SS systems, discussion of intergenerational equity consequences has tended to be constrained in the following ways: First, only two polar SS arrangements (pay-as-you-go and fully funded) are considered; second, a certain narrowly defined use of the SSTF (Case I) is assumed; and, third, some variant of SS actuarial status is used to measure intergenerational equity. These constraints often lead to the following view regarding intergenerational equity consequences: a pay-as-you-go SS arrangement is an income redistribution scheme across generations while a fully funded SS arrangement breaks this intergenerational link. Obviously, this view does not capture the potential intergenerational equity consequences which can exist when a wide range of SS arrangements and varying government uses of the SSTF are considered.

For instance, consider an imaginary study of the U.S. SS system in which the SSTF is assumed to yield the rate of return on physical capital (Case I). Noting that the actual U.S. SSTF is saved in the form of U.S. Treasury Securities (U.S. government bonds), the assumption necessarily implies that the U.S. government expenditure of the revenues accrued from issuing these securities is included as physical capital investment in the U.S. national account. In short, the view posits that the U.S. government bonds are

capitalized into an equity claim. Is this the case with the real U.S. economy? Could the intergenerational equity consequences derived in the study be the U.S. experience? Our approach can be understood in the following context: We modify the imaginary study in such a way that it captures other possible U.S. government uses of the SSTF, including a further differentiation of its capitalization method.[18]

Question 5: *If there is any meaningful relationship(s) between the size and use of the*

SSTF and intergenerational equity, how does it depend on structural aspects

of the model economy such as the population growth rate, labor share, and

consumption time preference?

For instance, there is a notion that a decrease in the population growth rate is unfavorable for a less funded SS arrangement. A lower population growth rate decreases the ratio of retired population to working population (the dependency ratio), and, as the population growth rate decreases, the implicit rate of return on less funded SS taxes tends to be less attractive. When this comparison is restricted to the two polar SS arrangements, pay-as-you-go and fully funded, the notion indicates that, for a lower population growth rate, a fully funded SS arrangement is preferred to pay-as-you-go.

This may not necessarily the case with our economic model. This is mainly because a change in the population growth rate alters equilibrium relative prices such as

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[18] For a detailed account of this, see, for example, Eisner (1989) and Blanchard and Fischer (1989). A similar point is discussed in Kotlikoff (1992). Eisner recapitulates some principal facts in the U.S. national accounting system, one of which is that part of the U.S. debt is capitalized into, for example, human capital.

the real wage rate and the rate of return on physical capital by changing the ratio of physical capital to (effective) labor. As Blanchet and Kessler (1991) indicate, excessive funding, coupled with an increase of the physical capital/(effective) labor ratio, may lead to a decreasing rate of return on physical capital; thus, a more funded SS arrangement is not necessarily preferable.

3.2. Computation of an Equilibrium Allocation

To operationalize our economic model, we first need to specify values for $(K_1^p, K_1^g, H_1, N_0, n)$. Throughout all simulations except those in Section 3.8, the following values are maintained. The size, N_0, of the population of generation 0 old agents in the initial period 1 is set at 1. The net population growth rate, n, is taken to be .012, so that the population is increasing over time. The size, N_1, of the population of generation 1 young agents in period 1 is then given by $(1 + n)$. The private physical capital stock in period 1 is given by .25 and is assumed to be purely owned by the generation 0 old agents. The government physical and human capital stocks in period 1 are each assumed to be 0, indicating that the SS system is implemented at the beginning of period 1. The efficiency, e_1, in period 1 is then given by 1 since no government human capital investment takes place in period 1.

We arbitrarily choose the following values for the parameters $(\alpha, \beta, A, \lambda)$ characterizing the production and utility functions. The share of physical capital in production, α, is set at .3. The technology factor, A, is taken to be 1, implying that there

is no technological progress. The consumption time preference in the utility function, β, is set at .5 meaning that an economic agent in our economic model is indifferent to the timing of his consumption as long as the present value of his consumption remains unchanged. Finally, we examine two different values of efficiency, 1 and 3, for the efficiency parameter λ. When the value is 1, the productivity of human capital is low; when the value is 3, it is high.

Given the above initialization and parameterization of our economic model, we compute the equilibrium allocation through an algorithmic process. This process turns out be simple because, as seen in Chapter 2, the equations describing the dynamic equilibrium allocation of our model reduce to a single basic causal difference equation (2-48). Using this equation, the time path of the physical capital/(effective) labor ratio is obtained, and, once this is done, equilibrium values for all the other endogenous economic variables (in each relevant period) can be derived by straightforward calculation. It should be particularly noted that the efficiency level e_{t+1} in period $t+1$ can be expressed as a function of per capita physical capital in period t.

For a given configuration of the initial conditions and parameters, a given SS arrangement (τ, δ), and a given government fiscal policy (a_g, a_k), a collection of sequences $\left\{c_t^y, c_t^o, s_t^p\right\}_{t=1}^{t=\infty}$, $\left\{v_t, \text{SSTF}_t, G_t, T_t, S_t^g, b_t\right\}_{t=1}^{t=\infty}$, and $\left\{K_t^p, K_t^g, K_t, H_t, N_t, L_t, r_t, \widetilde{w}_t, w_t\right\}_{t=1}^{t=\infty}$ is computed using the following algorithm:

Step 1: Start with the particular set of initial conditions and the particular values for the parameter values characterizing the production and utility functions described above.

Step 2: Set admissible values for the SS arrangement parameters, τ and δ, and the government fiscal policy parameters, a_g and a_k.

Step 3: Obtain the per capita physical capital in period 1 and the efficiency per unit of raw labor in period 2 as follows:

$$k_1 = \frac{\overline{K}_1^p + \overline{K}_1^g}{L_1} = \frac{\overline{K}_1^p}{e_1(\overline{H}_1, \overline{N}_0)N_1} = \frac{\overline{K}_1^p}{(1 + n)\overline{N}_0};$$

$$e_2 = e_2(k_1) = \exp^{\lambda(1 - a_k)(1 - a_g)\tau\delta(1 - \alpha)k_1}.$$

Step 4: Use equation (2-48) to calculate the equilibrium values for k_2 in period 2 and the efficiency level e_3 for period 3.

Step 5: Using relevant conditions appeared in the equilibrium presentation (Section 2.8), derive the corresponding equilibrium values for the endogenous variables.

Step 6: Repeat steps 4 and 5 with k_3 in place of k_2 and e_4 in place of e_3.

Step 7: Repeat Step 6 for successive values k_t and e_{t+1} until a designated period is reached or until convergence to a steady state allocation is achieved.

3.3. Measures of Welfare Benefit and Intergenerational Equity

Given any social security (SS) system, let its arrangement parameters (τ, δ) and government fiscal policy parameters (a_g, a_k) be abbreviated by $\Gamma = (\tau, \delta)$ and $\Phi = (a_g, a_k)$. Our measure of the welfare benefit WB_t associated with switching from one SS arrangement (Γ_0, Φ_0) to another arrangement (Γ_1, Φ_1) in a particular period t is defined as the (normalized) utility that a generation t young agent gains or looses by the switch:

$$WB_t ((\Gamma_0, \Phi_0), (\Gamma_1, \Phi_1)) = U_t^* (\Gamma_1, \Phi_1) - U_t^* (\Gamma_0, \Phi_0), \qquad (3\text{-}1)$$

where $U_t^* = 100 \cdot U_t + 200$. The reason for using normalized utility is clarified below.

As the equity debate in an intergenerational context attracts growing attention, the term "intergenerational equity" is used more frequently. To avoid confusion, it is important to provide a quantitative definition of the term. One possibility is to assume that the intergenerational equity aspect of a SS system is evaluated by whether or not the SS benefits received by each generation exceed the SS taxes they paid, in present value terms, and by how much. For each generation t, define θ_t to be the ratio of the present value of the SS benefits received in period $t + 1$, $b_{t+1}/(1 + r_{t+1})$, to the SS tax payment in period t, τw_t, for a representative generation t young agent:

$$\theta_t = \frac{b_{t-1}(1 + r_{t-1})}{\tau w_t}. \tag{3-2}$$

The plot of θ_t over time then provides a way of assessing intergenerational equity. If θ_t is greater than 1, the SS system is said to be actuarially favorable for generation t, and if less than 1, unfavorable. When θ_t is one, the SS system is said to be actuarially fair for generation t. Once the time profiles of θ_t associated with different SS systems are obtained, we can evaluate the extent to which a switch in systems improves the actuarial status of generations $t \geq 1$.

An alternative measure of intergenerational equity will now be defined in terms of lifetime utility. Let RWB_t denote the relative welfare benefit that a generation t young agent gains or losses by a switch in SS systems:

$$RWB_t((\Gamma_0, \Phi_0), (\Gamma_1, \Phi_1)) = \frac{U_t^*(\Gamma_1, \Phi_1) - U_t^*(\Gamma_0, \Phi_0)}{U_t^*(\Gamma_0, \Phi_0)}, \tag{3-3}$$

where the use of normalized utility ensures that the denominator is always positive for the range of parameter values used in this study and, during the course of the entire simulations, the same base case is used to derive $U_t^*(\Gamma_0, \Phi_0)$; namely, a particular parameterized version of the economic model in which there is no SS system.

The time profile of RWB_t then provides an alternative way to measure intergenerational equity. In particular, given a switch in SS systems, one can use this time profile to see which generations experience a gain in lifetime utility and which a loss.

3.4. Unfunded Social Security Tax Policy and Actuarial Status

The economic model in the absence of a SS system was first run as a base case. The main results of the base case run are reported in Table 3.1. Immediately apparent from Table 3.1 is that the physical capital/labor ratio decreases over time in a monotone fashion until its stationary value, .2194, is reached. The monotone decrease of per-capita physical capital stock up to the stationary value can be explained partly by the complete depreciation assumption and partly by the choice of initial physical capital stock.

Given the monotone decrease of the per capita capital stock, the corresponding time paths of the (effective) wage rate, the rate of return on the physical capital, and per-capita GNP are monotone as well. Up to the respective steady state values, .4441, -.1326, and .6344, the effective wage rate decreases, the return rate on the physical capital increases, and per-capita GNP increases in a monotone fashion. The absence of a SS system in the basic model economy implies that there exists no human capital investment. As a result, in each period $t \geq 1$ both the wage rate and the effective wage rate are the same, and the effective labor supply is equal to the generation t population. These obvious observations are not reported in Table 3.1.

Table 3.1: Equilibrium of the economic model in the absence of a social security system
--- $\tau = 0$, $n = .012$, $A = 1.0$, $\alpha = .3$, $\beta = .5$

Period	N_t	k_t	r_t	w_t	s_t^p	c_t^y	c_{t-1}^o	y_t	GNP_t	U_t	U_t^*
1	1.012	.2470	-.2016	.4602	.2301	.2301	.1947	.6574	.6653	-1.553	44.71
2	1.024	.2274	-.1539	.4489	.2244	.2244	.1932	.6412	.6646	-1.569	43.10
3	1.036	.2218	-.1390	.4455	.2228	.2228	.1928	.6365	.6676	-1.574	42.61
4	1.049	.2201	-.1345	.4445	.2223	.2223	.1927	.6350	.6741	-1.575	42.47
5	1.061	.2196	-.1332	.4442	.2221	.2221	.1926	.6346	.6817	-1.576	42.42
6	1.074	.2195	-.1327	.4441	.2221	.2221	.1926	.6345	.6897	-1.576	42.41
7	1.087	.2194	-.1326	.4441	.2221	.2221	.1926	.6344	.6980	-1.576	42.40
8	1.100	.2194	-.1326	.4441	.2221	.2221	.1926	6344	.7063	-1.576	42.40
9	1.113	.2194	-.1326	.4441	.2220	.2220	.1926	6344	.7148	-1.576	42.40
10	1.127	.2194	-.1326	.4441	.2220	.2220	.1926	6344	.7234	-1.576	42.40

For the steady state per-capita physical capital value, however, GNP is a non-stationary with a deterministic trend which is equal to the gross population growth rate, (1+n), as can be verified through simple mathematics using the economic model equations. In each period t, the consumption level of generation t young consumer is equal to the present value of his consumption level when old due to the equal weight given to each period's consumption decision ($\beta = .5$). The only reason why the old consumption level increases while the young consumption level decreases up to their stationary values is that, as the per capita physical capital stock decreases over time, the rate of return on physical capital increases.

Not surprisingly, but difficult to explain, the lifetime utility of the generation t young agent decreases with t until its stationary value, 42.40, is reached. The explanation for this result depends on general equilibrium relative price effects. For this reason, the detailed interpretation of this result is put aside until after relative price effects have been examined. A general intuitive explanation however can be given as follows: In this

particular run, detrimental effects from the decrease of the wage rate due to the decrease of the per capita physical capital stock may dominate beneficial effects from the increase of the return rate on physical capital due to the decrease of per capita physical capital stock and the corresponding increase of individual saving. Notice that the lower level of the per capita physical capital stock does not necessarily lead to lower lifetime utility. The decrease in lifetime utility is due partly to the choice of a logarithmic utility function, and partly to the choice of the initial capital stock.

It is useful to look at the time-series data in Table 3.1 graphically, especially since the economic model happens to be deterministic. As Kydland and Prescott (1996, p. 75) explain, this is mainly because what is mostly relevant in this deterministic economic model lies in the comparison of one equilibrium *path* with another. Figure 3.1 shows 10 observations of per-capita physical capital stock covering period 1 to period 10 for $\tau = 0$. The vertical axis measures the per capita physical capital stock and the horizontal axis measures time. In the absence of a SS system ($\tau = 0$), the decrease in the per-capita physical capital stock is seen to be strictly monotone. Likewise, the time path of gross national product (GNP), the net rate of return on physical capital, and the implicit rate of return on SS tax contributions are also depicted in Figure 3.1(b), 3.1(c), 3.1(d), respectively. One important implication of Figure 3.1 is that, for the given population growth rate, the introduction of a SS system does not change a basic property of equilibrium paths in per capita form: namely, their convergence to a steady state. A careful look at the basic causal difference equation (2-48) explains why.

(a) Physical capital/labor ratio (k)

(b) Gross national product (GNP)

(c) Net rate of return on physical capital (r)*   (d) Implicit rate of return on SS tax (b/v)**

* Horizontal axis meets vertical axis at steady state golden rule net rate of return on k, r* = .012.
** Horizontal axis meets vertical axis at the gross population growth rate (1 + n) = 1.012.
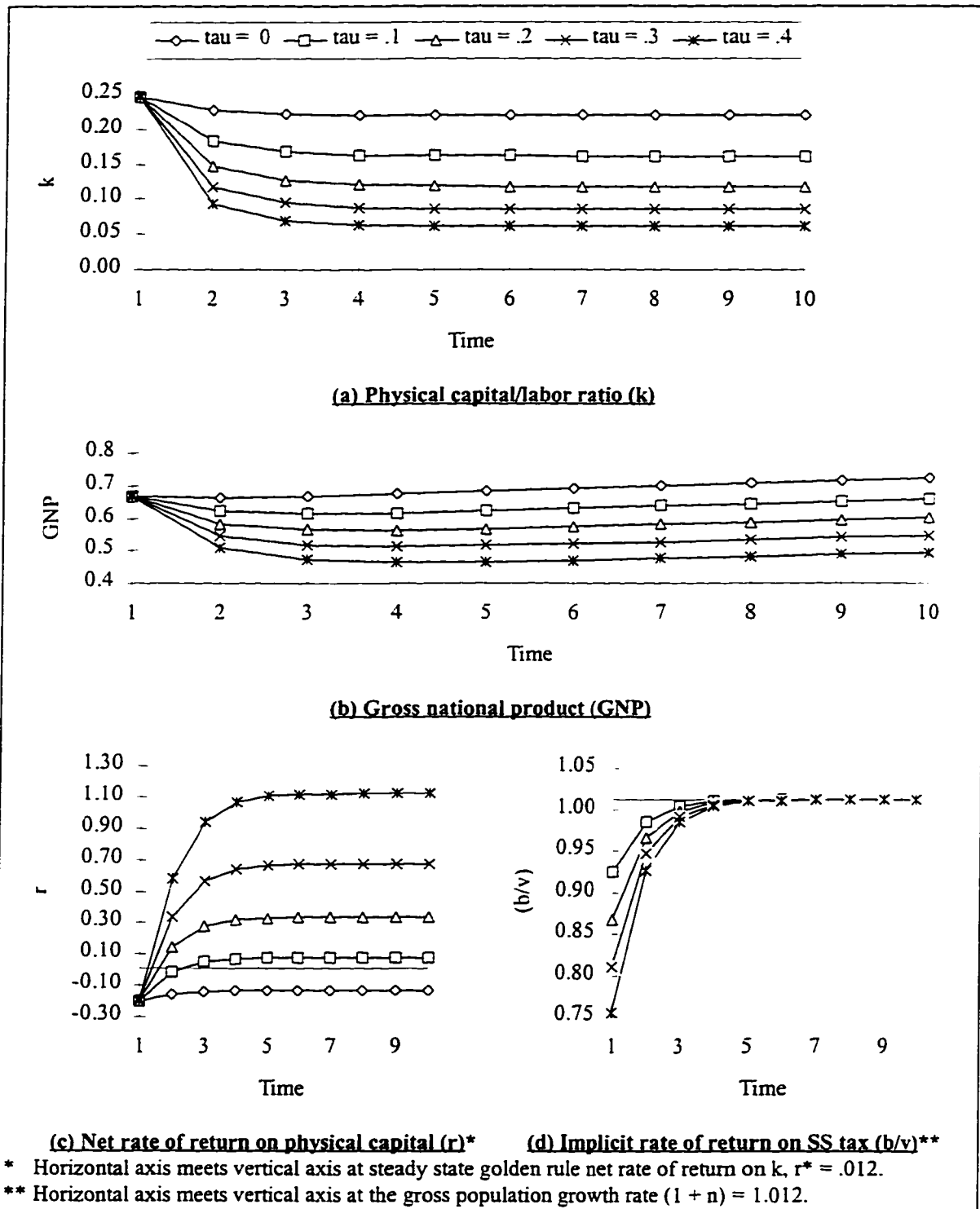
Figure 3.1:   Time paths of key variables as the SS tax policy τ changes 0 to .4
          ---δ = 0; n = .012, A = 1.0, α = .3, β = .5

Simulations were then conducted to determine how different SS tax rates τ in a pay-as-you-go SS system ($\delta = 0$) influence the economic model, particularly, the time profiles of intergenerational equity measures. Table 3.2 summarizes the economic model's steady state responses to changes in the SS tax rate τ for the τ values $\{0, .1, .3, .4\}$. The time paths of the per capita physical capital stock and gross national product (GNP), in addition to two rates of return, are reported in Figure 3.1. Time profiles of SS actuarial status θ_t and lifetime utility are depicted in Figure 3.2.

A relatively small SS tax rate ($\tau = .1$) decreases generation t private saving in each period t, which should be considered as one of the most important economic aspects of these simulations for a pay-as-you-go SS system. The result itself seems to be conceptually obvious; part of generation t private saving in period t is replaced by SS benefits in period t + 1 which is directly transferred from SS tax contributions made by generation t + 1 young agents. It does not, however, necessarily imply that its explanation is consequently easy, particularly in this general equilibrium context.

Upon the implementation of a pay-as-you-go SS system with $\tau = .1$, reduced disposable income in period t and a new source of old age income, SS benefits in period t+1, both have depressing effects on generation t saving in period t. However, if these depressing effects result in a reduced aggregate saving level in each period t, then the resulting rise in the rate of return on physical capital in period t + 1 tends to encourage generation t young personal saving in period t, and the resulting fall in wage rate in

Table 3.2: Steady state equilibrium and intergenerational equity for the economic model under alternative unfunded SS policies τ

---n = .012, A = 1.0, α = .3, β = .5

Variable	$\Gamma_0(.0, .0)$	$\Gamma_1(.1, 0)$	$\Gamma_2(.2, 0)$	$\Gamma_3(.3, 0)$	$\Gamma_4(.4, 0)$
$\bar{k}$	.2194	.1612	.1182	.0859	0.061
$\bar{r}$	-.1326	.0763	.3373	.6729	1.120
$\bar{w}$	.4441	.4049	.3689	.3351	.3028
$\bar{v}$	0	.0405	.0738	.1005	.1211
$\bar{b}$	0	.0410	.0747	.1018	.1226
$\bar{s}^p$	.2220	.1632	.1196	.0869	.0619
$\bar{c}^y$	.2220	.2012	.1755	.1477	.1197
$\bar{c}^o$	.1926	.2166	.2347	.2471	.2539
$\bar{y}$	.6344	.5784	.5270	.4788	.4325
$\bar{U}$	-1.576	-1.567	-1.595	-1.655	-1.747
$\bar{U}*$	42.40	43.35	40.51	34.48	25.34
$\overline{WB}$	0	.945	-1.892	-7.920	-17.070
$\bar{b}/\bar{v}$	-	1.012	1.012	1.012	1.012
$\bar{\theta}$	-	.9403	.7568	.6049	.4773
$\overline{RWB}$	0	.0223	-.0446	-.1868	-.4026

(a) Social security actuarial status (θ)

(b) Normalized Lifetime Utility (U*)
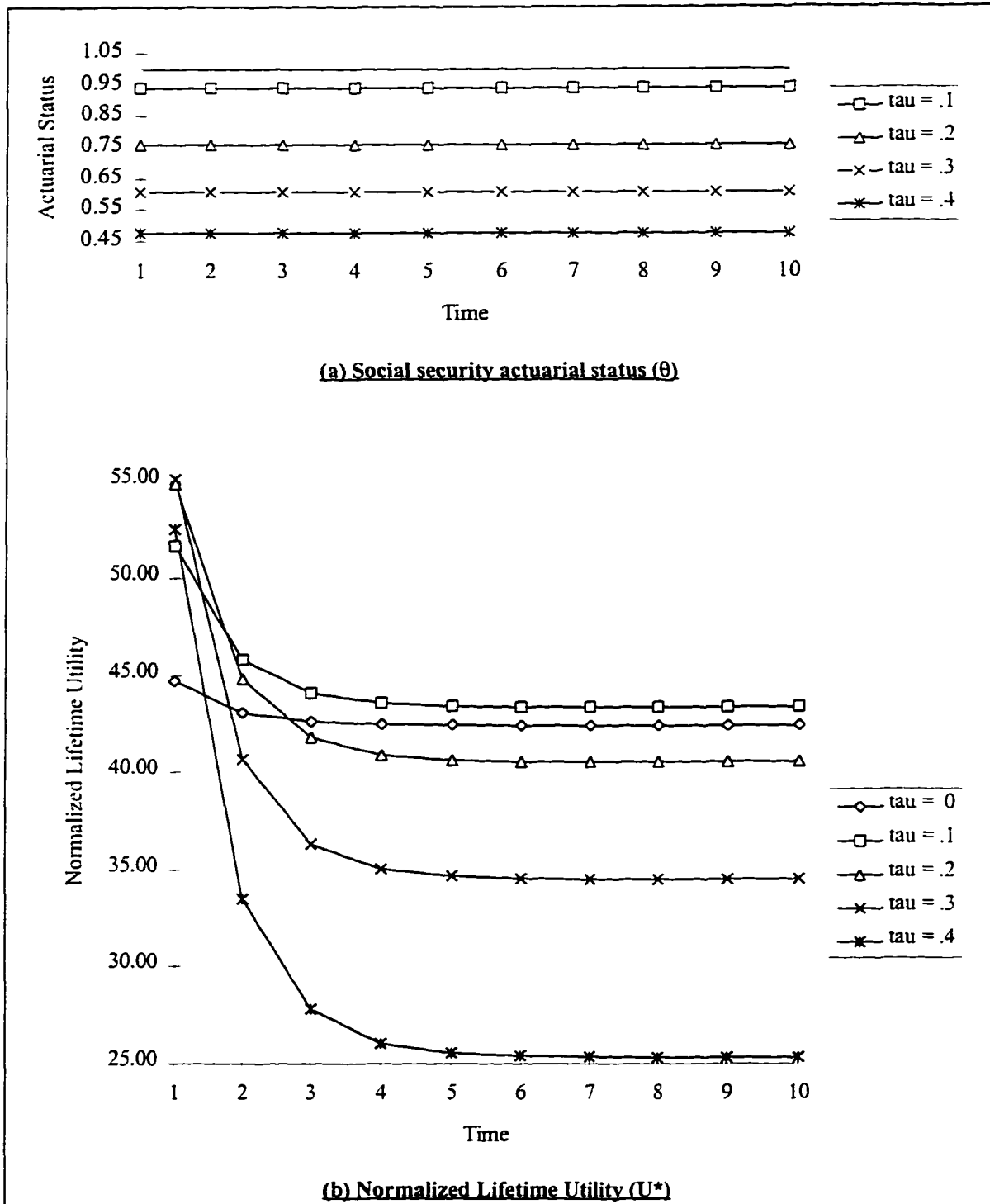
Figure 3.2: Time profiles of welfare and social security actuarial status as the SS tax
policy τ changes from 0 to .4
---δ = 0; n = .012, A = 1.0, α = .3, β = .5

period t + 1 has a similar effect on the private saving through a reduction of the SS

benefits in period t + 1.

However, as long as equilibrium relative prices are time-invariant, for instance, in

a steady state solution, the explanation is clear-cut. By definition, steady state individual

saving $\bar{s}^p$ is the steady state per capita physical capital stock multiplied by the population

growth rate, $\bar{s}^p = (1 + n)\bar{k}$, under a pay-as-you-go SS system. In order to ascertain the

impact of variations in SS tax rate, τ, on steady state per capita physical capital stock, $\bar{k}$,

it is enough to conduct a comparative static analysis of the change in $\bar{k}$ with respect to τ.

The sign of $d\bar{k}/d\tau$ is unambiguously negative.[19] Consequently, $\bar{s}^p$ decrease as well.

Furthermore, noting from Figure 3.1 that the time paths for the per capita physical capita

stock corresponding to different τ values are monotone decreasing and do not intersect

each other, we can verify that the pay-as-you-go SS system results in a crowding out of

both saving and per capita physical capital formation over time for the indicated

maintained parameter values.

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[19] When the SS system is arranged on a pay-as-you-go basis ($\delta = 0$), $\bar{k}$ is given by $\bar{k} = \left(\dfrac{C'}{D'}\right)^{\frac{1}{1-\alpha}}$ where

$C' = A(1 - \alpha)(1 - \beta)(1 - \tau)$ and $D' = (n + 1)\left(1 + \dfrac{1 - \alpha}{\alpha}\beta\tau\right)$. Then, we have

$$\frac{d\bar{k}}{d\tau} = \frac{1}{(\alpha - 1)}\left(\frac{D'}{C'}\right)^{\frac{-\alpha}{(\alpha-1)}}\left[\frac{C'\frac{\partial D'}{\partial \tau} - D'\frac{\partial C'}{\partial \tau}}{C'^2}\right]^{\frac{-\alpha}{(\alpha-1)}} < 0, \text{ noting that}$$

$\partial C'/\partial \tau = -A(1 - \alpha)(1 - \beta) < 0$, $\partial D'/\partial \tau = (1 + n)\dfrac{1 - \alpha}{\alpha}\beta > 0$ and $(\alpha - 1) < 0$.

Once the nature of the time path for the per capita physical capital stock is explained, there is no surprise regarding the time variation of GNP: The level of GNP decreases monotonically over time but constancy of the physical capital/labor ratio in steady state implies that GNP grows at the same rate, $(1 + n)$. In short, the economic growth rate is independent of τ. That is, τ has an only level effect. The introduction of a pay-as-you-go SS system with a relatively small SS tax rate ($\tau = .1$), however, increases lifetime utility in each period t relative to the base case with no SS system, implying that this particular arrangement is dynamically Pareto improving. It is tempting to conclude from these observations that a pay-as-you-go SS arrangement tends to resolve a problem of over-accumulation in the per capita physical capital stock. Although there is nothing wrong with this explanation, it could be potentially misleading in the sense that, for other parameter values, it might also have caused a problem of under-accumulation in the per-capita physical stock. For this reason, we interpret, for the moment, the result as follows; balancing out positive and detrimental relative price effects, the economy as a whole can benefit from the introduction of a pay-as-you-go SS system with a relatively small SS tax rate ($\tau = .1$).

Consider, instead, the following question: How is it possible that the lifetime utility of each generation t agent increases with the introduction of a SS system with $\tau = .1$ while his income in period t decreases and per-capita GNP decrease in both in period t and $t + 1$? Are these two facts reconcilable with each other? Although this question obviously scales down the question of why Pareto improvement results with the

introduction of the SS arrangement with $\tau = .1$, answering the question may provide a clue regarding this Pareto improvement. Let us take a look at the beneficial effects of the decrease in the per capita physical capital stock. One obvious beneficial effect is that, as the rate of return on individual saving rises, the generation t old agent's income in period $t + 1$ increases. One detrimental effect is that the fall in the wage rate in period t directly depresses the generation t young agent's disposable income.

There is one more unsettling effect, which is determined by comparing the rate of return on physical capital in period $t + 1$ to the rate of return on the SS tax contributions of the generation t young agent in period t. [Note that, in a pure exchange overlapping generations economy, this comparison is irrelevant because of the absence of physical capital.] The latter rate of return is given by the SS benefits received in period $t + 1$ divided by the taxes paid in period t. The time path of (b_{t+1}/v_t) is depicted in Figure 3.1(d). The implicit rate of return on SS tax payments in period t is simply the population growth rate multiplied by the wage growth rate between period t and $t + 1$, which, in steady state, is equal to the biological growth rate. Notice that, when $\delta = 0$ and the SS actuarial status θ_t in period t is multiplied by the gross rate of return on physical capital in period $t + 1$, it becomes the implicit rate of return.

Consequently, if θ_t is greater than one, then saving through the SS system dominates personal saving in the sense of providing a higher rate of return, implying that θ_t has a positive effect on lifetime utility. If θ_t is less than one, the reverse conclusion holds. The effects on θ_t of alternative SS systems are given in Table 3.2 for the steady

state case and Figure 3.2(a) for the dynamic case. The time profiles for θ_t in Figure 3.2(a) look strange because the value of θ_t remains constant over time.[20] The point is, the value of the actuarial status θ_t for a pay-as-you-go SS system with $\tau = .1$ is always less than one, implying that saving through SS tax payments is dominated by private saving and thus has a negative effect on each generation t agent's lifetime utility in each period t. However, this lack of dominance is not enough to establish that the introduction of the SS system to an economy currently without a SS system would be harmful because it ignores the effects of the introduction on physical capital accumulation and hence on the wage rage w_t and the rate of return on physical capital r_t. Indeed, as shown in Figure 3.2(b), lifetime utility is actually higher in every period t for $\tau = .1$ than $\tau = 0$.

Now we can reconcile the seemingly contradictory facts that the lifetime utility of a generation t consumer increases in moving from $\tau = 0$ while his income when young

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[20] As far as a pay-as-you-go SS arrangement is concerned, the time-invariance of θ_t for a given τ can be verified analytically. That is, for a given $\delta = 0$, θ_t is given by

$$\theta_t = \frac{b_{t+1}}{v_t(1 + r_{t+1})} \quad \text{(by (3-2))}$$

$$= \frac{(1 + n)w_{t-1}}{(1 + r_{t+1})w_t} \quad \text{(by (2-6) and (2-13))}$$

$$= \frac{(1 + n)(1 - \alpha)k_{t-1}}{\alpha k_t^\alpha} \quad \text{(by (2-25) and (2-34))}$$

$$= \frac{(1 + n)(1 - \alpha)}{\alpha} \frac{C}{D}. \quad \text{(by (2-68))}$$

Here noting that both C, (2-69), and D, (2-70), are constant over time we can easily see the constancy of a pay-as-you-go SS actuarial status over time.

and per-capita GNP decrease. These are reconcilable as long as the beneficial effects of the decrease in the per capita physical stock, such as the rise in the rate of return on individual saving, dominate the detrimental effects, such as the decrease in the wage rate and the lower rate of return on SS tax payments relative to the rate of return on physical capital. The simulation results for a SS arrangement with $\tau = 0$ show that this is the case for the particular parameter configurations studied in Figure 3.2. It is not, however, particularly important to our study whether these facts can co-exist or not. What draws our attention most is that the introduction of a pay-as-you-go SS system with $\tau = .1$ has, on the one hand, a detrimental effect on individual welfare from a static "within SS system" perspective, while on the other hand it is also a Pareto improvement. The fact that the introduction of a Pareto improving pay-as-you-go SS system, even with a positive population growth rate, can result in an SS actuarial loss, has important political implications, since the political popularity of a change in a SS system often depends on its effects on SS actuarial status.

To explore this point further, we simulate different SS tax schemes in the context of a pay-as-you-go system ($\delta = 0$). Table 3.2 displays steady state simulation results. Note in particular that the change in the SS tax rate affects the magnitude of the steady state gain or loss in welfare benefits WB,

$$\overline{WB}(\tau = .1) > \overline{WB}(\tau = 0) > \overline{WB}(\tau = .2) > \overline{WB}(\tau = .3) > \overline{WB}(\tau = .4).$$

This ordering provides important evidence bearing on the question of why the introduction of the SS system with tax rate $\tau = .1$ is welfare improving, at least in the context of the steady state. The SS tax scheme with $\tau = .2$ turns out to be welfare-deteriorating compared to the base case $\tau = 0$ with no SS system and to the case $\tau = .1$. This clearly suggests that, in steady state, the economy experiences an over-accumulation of physical capital in the absence of a SS system and an under-accumulation of physical capital in a pay-as-you-go SS system with $\tau = .2$. In particular, it appears that the introduction of a pay-as-you-go SS system with $\tau = .1$ alleviates the problem of physical capital over-accumulation process, without pushing the economy so far as to cause a severe under-accumulation problem.

Then, what specification of the SS tax rate would resolve the capital accumulation problem in our economic model completely for the current set of parameter values? Or, in short, what is the optimal SS tax rate in the current setting? As is widely-known, it depends on the choice of generational welfare function. In order to avoid this choice problem, we will arbitrarily suppose that the government's objective in implementing a SS system is to ensure a Pareto optimal steady state allocation for the economy, which in turn requires that this steady state allocation be the golden rule allocation. As discussed in Chapter 2, this golden rule allocation is achieved if and only if the net rate of return on physical capital is equal to the net population growth rate n.

Using (2-69) (Feature 1 in Section 2.12), the Pareto optimal SS tax policy can be obtained as a matter of calculation. For the currently maintained parameter values, it is,

approximately, $\tau^* = .0714$. Indeed, the Pareto optimal SS tax rate explains the simulation results of lifetime utility gain or loss over different specifications of the SS tax rate. That is, a SS tax rate $\tau = .1$ is the closest tested tax rate to the Pareto optimal SS tax rate. However, although the pay-as-you-go tax scheme with $\tau = .1$ leads to the highest level of steady state lifetime utility, it needs to be decreased slightly down to ensure a steady state golden rule allocation. In other words, under-accumulation of the steady state per capita physical capital stock occurs at $\tau = .1$ and needs to be corrected by lowering the SS tax rate a little bit. Specifically, the net rate of return on physical capital with $\tau = .1$, $.07625$, is higher than the golden rule rate, which is given by the net population growth rate $.012$.

The time profile of actuarial status θ_t shows whether or not a SS system is favorable from a 'within SS system' view. As Figure 3.2(a) shows, a lower SS tax rate yields a higher value of θ_t in each period t:

$$\theta_t \, (\tau = .1) > \theta_t(\tau = .2) > \theta_t(\tau = .3) > \theta_t(\tau = .4).$$

The ordering is explained by the fact that an increase in SS tax rate crowds out per capita physical capital stock. In steady state, relative prices remain constant over time and, thus, the only relevant variables for the determination of θ_t are the population growth rate and the rate of return on physical capital. As the SS tax rate increases, the rate of return on physical capital increases, too, due to the crowding out of per capita physical capital stock. For the given exogenous population growth rate, this implies that the SS system

becomes less favorable as the SS tax rate increases. Moreover, Figure 3.2(a) reveals that the SS tax contribution over time is in any case dominated by personal saving in terms of rate of return; hence, measured solely in terms of SS actuarial status, *all* the SS arrangements here are unfavorable.

As indicated in Table 3.2, in steady state, $\bar{\theta}$ and $\overline{WB}$ give the same ranking for SS tax schemes. However, an actuarially unfavorable SS arrangement with $\tau = .1$ yields a higher level of lifetime utility than the case $\tau = .1$ with no SS system. This point clearly raises a question whether or not actuarial status is an effective measure of the desirability of a SS system. As discussed, the size of θ_t may affect lifetime utility, but it is not the sole factor that affects lifetime utility. Indeed, the orderings of $\overline{WB}$ and $\bar{\theta}$ indicate that this is the case. This can be better explained in a dynamic context. In period 1, a higher level of lifetime utility for a generation 1 consumer does not necessarily correspond to a greater value of θ_1. That is, as Figure 3.2(b) shows, the ordering of WB_1 in period 1 is as follows;

$$WB_1(\tau = .3) > WB_1(\tau = .2) > WB_1(\tau = .4) > WB_1(\tau = .1) > WB_1(\tau = .0).$$

It is often conceived that one of the potential gains of a pay-as-you-go SS system is the so called "positive growth dividend," meaning that, if there are more workers than retirees, the implicit net rate of return on SS tax payments is positive: A positive population growth rate enables each agent to receive larger benefits when old than what

he paid in as SS taxes when young. This view is, first, too simple, and, second, even erroneous. Figure 3.2(a) shows that, as the wage rate decreases due to the crowding out effect of a positive SS tax rate on per capita physical capital (namely, the wage congestion effect), the net rate of return on SS tax payments could be negative: Even with an increasing population economy, the present value of SS benefits tends to be less than the present value of SS tax payments.

An overall implication of the simulation results is that a positive population growth rate may not necessarily make a pay-as-you-go SS arrangement more preferable to other types of SS arrangements. The reason for this cannot be seen from looking only at SS actuarial status θ_t: It is due to the fact that a pay-as-you-go SS arrangement may not have any particular advantage for tuning the per capita physical stock to an appropriate level. This again, raises the question of whether or not SS actuarial status θ is an effective measure of intergenerational equity. The simulation results illustrate why SS actuarial status is not sufficient to reflect the gain or loss of lifetime utility associated with a switch from one SS system to another.

3.5. Social Security Funding Policy and Intergenerational Equity

Case I: $a_g = 0$ and $a_k = 1.0$

So far we have been considering pay-as-you-go SS arrangements ($\delta = 0$). From the simulation results, we are led to doubt that actuarial status θ is an effective measure of intergenerational equity for such arrangements. We, now, have the task of considering

alternative SS funding policies δ and their resulting consequences for intergenerational

equity measured in terms of θ and RWB. First the SSTF is assumed to be invested

entirely as physical capital in the production technology (Case I: $a_g = 0$ and $a_k = 1.0$). The

simulations are rerun with different SS funding policies for a given SS tax policy $\tau = .2$.

The SS funding policy δ can vary from 0 (pay-as-you-go) to 1.0 (fully funded). Figures

3.3 and 3.4 summarize the simulation results obtained for dynamic equilibrium paths, and

Table 3.3 contains steady state equilibrium values of some important variables and

welfare measures.

Figure 3.3(a) clearly shows that an increase in δ, and hence in the relative size of

the SSTF, increases the per capita physical capital stock in each period t and, accordingly,

decreases its rate of return. This crowding-in of per capital stock due to the increase of the

SSTF fraction δ for the given government fiscal policy ($a_g = 0$, and $a_k = 1.0$) can be

explained in many ways. One is through a comparative dynamic analysis of the changes

in the level of $\bar{k}$ of per capital physical capital with respect to changes in δ, as was done

with respect to SS tax policy in Section 3.4. The other is through Feature 3 in Section

2.12. Feature 3 notes that, when $a_g = 0$ and $a_k = 1.0$, an increase in the SS funding policy

δ is equivalent to a decrease in the SS tax rate τ as long as the effective tax rate $\tau^* = \tau(1 - \delta)$ remains unchanged. Thus, the discussion in Section 3.4 there explaining why

crowding-out of the per capita physical capital stock results from an increase in the SS

tax rate τ also explains why an increase in δ raises the per capita physical capital stock.

(a) Physical capital/labor ratio (k)

(b) Net rate of return on physical capital*   (c) Implicit rate of return on SS tax**

* Horizontal axis meets vertical axis at steady state golden rule net rate of return on k, kabob .012.

** Horizontal axis meets vertical axis at the gross population growth rate $(1 + n) = 1.012$.
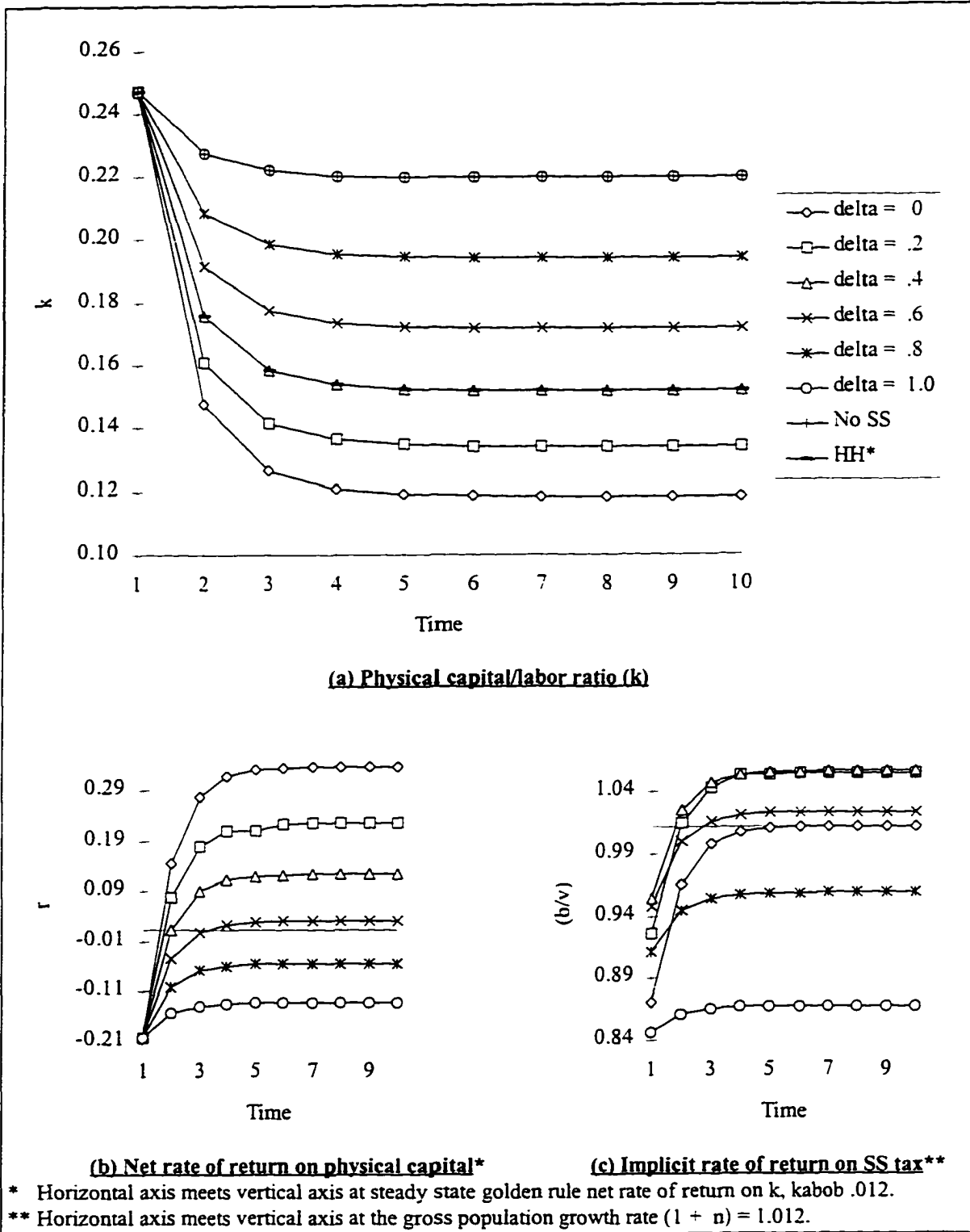
Figure 3.3:   Time paths of key variables as the SS funding policy δ changes from 0 to 1:
Case I ($a_g = 0$, $a_k = 1.0$)
---$\tau = .2$; $n = .012$, $A = 1.0$, $\alpha = .3$, $\beta = .5$

(a) Social security actuarial status (θ)*

* For a given δ, SS actuarial status (θ) is constant over time.

(a) Relative welfare benefit (RWB)
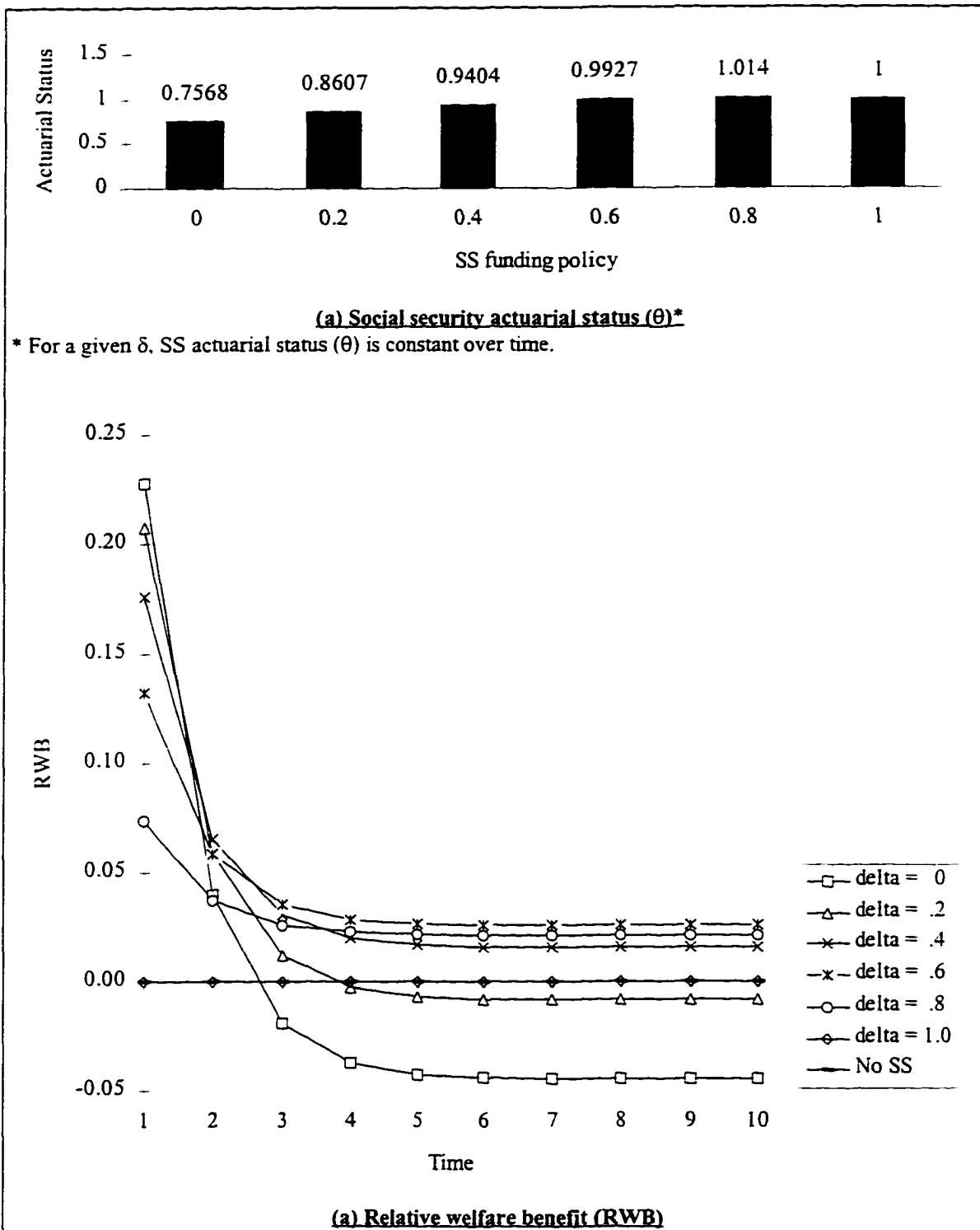
Figure 3.4: Time profiles of intergenerational equity measures as the SS funding policy
δ changes from 0 to 1: Case I ($a_g = 0$, $a_k = 1.0$)
---τ= .2; n = .012, A = 1.0, α = .3, β = .5

Table 3.3: Steady state equilibrium and intergenerational equity for the economic model under alternative SS funding policies δ: Case I ($a_g = 0$, $a_k = 1.0$) ---n = .012, A = 1.0, α = .3, β = .5

Variable	Γ(.2, .0)	Γ(.2, .2)	Γ(.2, .4)	Γ(.2, .6)	Γ(.2, .8)	Γ(.2, 1.0)	Γ(0, 0)
$\bar{k}$	.1182	.1339	.1516	.1715	.1939	.2194	.2194
$\bar{r}$	.3373	.2254	.1237	.0309	-.0543	-.1326	-.1326
$\bar{w}$	.3689	.3830	.3975	.4124	.4279	.4441	.4441
$\bar{v}$	.0738	.0766	.0795	.0825	.0856	.0888	0
$\overline{SSTF}^i$	0	.0153	.0318	.0495	.0685	.0888	0
$\bar{b}$	.0747	.0808	.0840	.0844	.0821	.0770	0
$\bar{s}^p$	.1196	.1202	.1216	.1240	.1278	.1332	.2220
$\bar{s}^g$	0	.0153	.0318	.0495	.0685	.0888	0
$\bar{s}^n$	.1196	.1355	.1534	.1735	.1963	.2220	.2220
$\bar{c}^y$	.1755	.1862	.1964	.2059	.2192	.2220	.2220
$\bar{c}^o$	.2347	.2281	.2207	.2123	.2029	.1926	.1926
$\bar{y}$	.5270	.5471	.5678	.6718	.6114	.6344	.6344
$\bar{U}$	-.1595	-1.580	-1.569	-1.565	-1.567	-1.576	-1.576
$\bar{U}*$	40.51	42.05	43.05	43.49	43.30	42.40	42.40
$\overline{WB}$	-1.89	-.36	.65	1.09	0.90	0	0
$\bar{b}/\bar{v}$	1.012	1.055	1.057	1.023	.9590	.8674	-
$\bar{\theta}$	.7568	.8607	.9404	.9927	1.014	1.000	-
$\overline{RWB}$	-.0446	-.0083	.0153	.0257	.0212	0	0

Figure 3.3(a) also provides an example illustrating why, when $a_g = 0$ and $a_k = 1.0$,

having no SS system ($\tau = 0$) is equivalent to having a fully funded SS system in the

economic model at hand. When the SSTF is used entirely for physical capital investment,

the introduction of a fully funded SS system is irrelevant because the time path of per

capita physical capital is not affected by this introduction. This is because agents do not

differentiate among types of saving as long as the saving yields them the same rate of

return (Feature 2 in Section 2.12). Figure 3.3(a) also shows that the two distinct SS

arrangements $\Gamma(.2, .2)$ and $\Gamma(.3, .6)$ yield the same time path for per capita physical

capital. Roughly put, a SS arrangement with low τ and low δ can be equivalent to a SS

arrangement with high τ and high δ. This happens whenever the effective SS tax rate

$\tilde{\tau} = \tau(1 - \delta)$ is the same for these SS arrangements (Feature 3 in Section 2.12). Note

that this effective SS tax rate is .12 for each of the SS arrangements $\Gamma(.2, .4)$ and $\Gamma(.3, .6)$.

The implicit rate of return on SS tax payments, defined as the ratio of SS benefits

received to SS taxes paid, is closely related to various economic rates such as the wage

and population growth rates and the rate of return on physical capital, in addition to the

SSTF fraction. The seemingly complex relationship between the implicit rate and other

economic rates can be understood in a piecemeal fashion.

When the SS system is run on a pay-as-you-go basis ($\delta = 0$), the SS benefits to

generation t old agents in period t + 1 are closely tied to the current working generation

through two factors: One is the population growth rate, and the other is the growth rate of

the real wage.[21] In short, the implicit rate of return on pay-as-you-go SS tax payments is given by $(b_{t+1} / v_t) = (1 + n)(w_{t+1} /w_t)$. When the SS system is run on a fully funded basis ($\delta = 1.0$), its implicit rate return rate in period $t + 1$ is equal to the rate of return on physical capital in period $t + 1$, implying that a fully funded SS system is always actuarially fair. More generally, the implicit rate of return on SS tax payments for any funding level δ, $0 \leq \delta \leq 1.0$, is given by

$$\frac{b_{t+1}}{v_t} = (1 - \delta)(1 + n)\frac{w_{t+1}}{w_t} + \delta(1 + r). \tag{3-4}$$

Under a partially funded SS system, when the SSTF fraction δ is large, the performance of the fund itself is the critical factor for determining the implicit rate of return. As the SSTF fraction δ gets smaller, the population and wage growth rates become the more critical factors. However, although the analytical expression for the implicit rate is simple, Figure 3.3 shows how its use can be complicated. Figure 3.3(b) and (c) reveal that a pay-as-you-go SS arrangement ($\delta = 0$) does not necessarily guarantee a higher implicit rate of return than other types of SS arrangements. As a matter of fact, there are three arrangements that dominate the pay-as-you-go SS arrangement in terms of implicit rate of return. The difficulty in using the implicit rate is that it is difficult to

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[21] For a detailed comparison of rates of return for an unfunded SS system, see, for example, Aaron (1966). For the case of a pure exchange economy, see Samuelson (1958) where the implicit (internal) rate of return on (SS) tax payments is equated to the growth rate of the population.

predict the relative effects of a switch in SS arrangements on wage growth and the rate of return of physical capital.

Figure 3.4 shows the time profiles of actuarial status θ_t and relative welfare benefit RWB_t for different SS funding policies δ. As for Case I, Figure 3.4(a) indicates that the economic rates respond to a given SS arrangement in such a way that the actuarial status for a given SS arrangement is time-invariant. There is only one SS arrangement with a value of θ that exceeds 1 and hence is actuarially favorable. All other arrangements are either actuarially unfavorable ($\delta < 1.0$) or fair ($\delta = 1.0$). The ordering is as follows:

$$\bar{\theta}(\delta = .8) > \bar{\theta}(\delta = 1.0) > \bar{\theta}(\delta = .6) > \bar{\theta}(\delta = .4)$$
$$> \bar{\theta}(\delta = .2) > \bar{\theta}(\delta = 0).$$

When evaluating intergenerational equity consequences of alternative SS funding policies δ from an SS actuarial status perspective, it is clear that the partially funded SS arrangement with $\delta = .8$ dominates the other SS funding policies over time, with the pay-as-you-go SS arrangement ($\delta = 0$) yielding the worst outcome. However, Table 3.3 reveals that this does not hold if intergenerational equity is instead evaluated from lifetime utility perspective.

Recalling the definition (3.1) for welfare benefit WB, the highest WB in steady

state is obtained when the SS funding policy δ is set at .6 instead of .8. The entire

ordering is given by

$$\overline{WB}(\delta = .6) > \overline{WB}(\delta = .8) > \overline{WB}(\delta = .4) > \overline{WB}(\delta = 1.0)$$
$$> \overline{WB}(\delta = .2) > \overline{WB}(\delta = 0).$$

On the other hand, recalling definition (3.3) for relative welfare benefit RWB, Figure

3.4(b) shows that the ordering of RWB in period 1 is

$$RWB_1(\delta = 0) > RWB_1(\delta = .2) > RWB_1(\delta = .4) > RWB_1(\delta = .6)$$
$$> RWB_1(\delta = .8) > RWB_1(\delta = 1.0).$$

Noting that the ordering of RWB in *any given period* is the same as the ordering of WB,

the ordering of RWB in period 1 indicates that the highest WB in period 1 occurs when δ

= 0, although the lowest steady state value of WB would be realized by this policy.[22]

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[22] It should be noted that the orderings of WB and RWB *across* periods do not necessarily coincide with each other. This is simply because counterfactual lifetime welfare in the RWB measure is different over time. For this reason, a comparison of WB across periods may not necessarily be appropriate, although WB is often used in comparing intergenerational welfare consequences. For instance, when WB_t is greater than WB_{t-1}, does it imply that generation t agents experience a higher degree of intergenerational equity than generation t - 1 agents do? Not necessarily, at least, according to the postulated measure, RWB. Suppose that there are three generations 1, 2, and 3 whose lifetime utilities, in the absence of SS system, are given by 1, 10, and 100, respectively. Upon the implementation of a SS system, suppose their lifetime utilities increase by .5, 5, and 50, respectively. The orderings of WB and RWB are then given, respectively, by,

$$WB(gen. 3) = 50 > WB(gen. 2) = 5 > WB(gen. 1) = .5,$$

More generally, Figure 3.4(b) indicates that there is, in any given period at least one SS funding policy that yields higher RWB (hence WB) than $\delta = .8$. Notice also, from Figure 3.4(a), that all of the SS funding policies except $\delta = .8$ and $\delta = 1$ are actuarially unfavorable ($\delta < 1.0$). These findings illustrate how the intergenerational equity consequences of a SS funding policy, when evaluated on an SS actuarial basis, can be misleading. A comparison of the orderings with respect to θ and WB clearly shows that SS actuarial status θ cannot be considered to be an effective measure of intergenerational equity: The measure θ is not sufficient to reflect the gain or loss in lifetime utility.

Now, let us take a closer look at the intergenerational equity consequences of alternative SS funding policies from a lifetime utility perspective. In the simulations for pay-as-you-go SS tax policy τ in Section 3.4, it was shown that there is a marked possibility of conflict across generations. This is again borne out in the simulations for the SS funding policy δ. As indicated in Table 3.3, the ordering of steady state RWB is as follows:

$$\overline{RWB}(\delta = .6) > \overline{RWB}(\delta = .8) > \overline{RWB}(\delta = .4) > \overline{RWB}(\delta = 1.0)$$
$$> \overline{RWB}(\delta = .2) > \overline{RWB}(\delta = 0).$$

---

and
$$RWB(\text{gen. 1}) = RWB(\text{gen. 2}) = RWB(\text{gen. 3}) = .5.$$

In this case, the orderings of WB and RWB are not the same.

However, from a time path perspective, Figure 3.4(b) shows that there is no SS funding policy that dominates all other funding policies in terms of RWB: Although the SS funding policy $\delta= .6$ yields the highest value of RWB in period 3 and after, this policy is dominated by the SS funding policy $\delta = .2$ in period 2 and by the SS funding policies $\delta= 0$, $\delta= .2$, and $\delta= .4$ in period 1. In particular, given a SS system with $\delta < 1.0$, there is no SS funding policy that increases the value of RWB for some generations without reducing it for at least one other generation. Notice, however, that, given the SS funding policy $\delta = 1.0$, there are three funding policies ($\delta = .4$, $\delta = .6$, and $\delta = .8$) that yield a higher value of RWB for all generations.

Is there any way to explain the ordering of steady state RWB over various SS funding policies? There is one, discussed in the previous section, although it fails to explain everything. It is shown there that tuning the per capita capital stock to an appropriate level might be key to obtaining a pay-as-you-go SS system that ensures the highest possible value for steady state RWB. More precisely, whether an optimal pay-as-you-go SS system is obtained in steady state completely depends on whether or not the steady state rate of return on per capita physical capital is equated to the population growth rate.

Feature 3 described in Section 2.12 implies that this reasoning is also applicable to the SS funding policy δ. Indeed, a comparison of the steady state rate of return $\bar{r}$ on physical capital to the steady state relative welfare benefit RWB verifies this (See Table 3.3.) That is, when $\bar{r}$ is close to $n = .012$, a higher value of steady state RWB is realized.

It should be noted that this is not applicable for non-steady state values. For instance, as seen in Figures 3.3(b) and 3.4(b), the rate of return on physical capital in period 2 that is closest to n = .012 occurs with a SS funding policy δ = .4, but δ = .2 yields a higher value of RWB for generation 1 agents.

The main conclusions of this section can be summarized as follows: First, SS actuarial status θ cannot be considered to be an effective measure of intergenerational equity. A comparison of Figure 3.4(a) with Figure 3.4(b) clearly shows that SS actuarial status does not accurately reflect the gains or losses in lifetime utilities. In some cases, an unfavorable SS actuarial status (θ < 1.0) and improved welfare (WB > 0) co-exist, implying that an evaluation of a SS system strictly on the basis of SS actuarial status can be misleading. This result will be shown in a far more decisive fashion in subsequent simulations. Second, no one SS funding policy dominates all other funding policies in terms of relative welfare benefit RWB. This shows how difficult it is to resolve potential conflicts among generations.

Most importantly, the conclusions obtained here are not very different from the ones drawn for the pay-as-you-go SS tax policy τ. This is because, as far as per-capita physical capital stock and intergenerational equity as measured via RWB are concerned, only the effective SS tax rate $\tilde{\tau}$ = $\tau(1 - \delta)$ matters. There are, of course, some different aspects, most of which are easily seen. First, the implicit rate of return on SS tax payments varies considerably over different types of SS arrangements $\Gamma(\tau, \delta)$, which may constitute one of the reason why the type of SS arrangement $\Gamma(\tau, \delta)$ is often considered to

be a vital matter. Second, as far as the generation 0 old in period 1 are concerned, the SS

funding policy δ critically matters even when the effective SS tax rate remain unchanged:

The generation 0 old always prefer a higher SS tax policy τ to a lower one and a lower SS

funding policy δ to a higher one.

However, the generation 0 old agents' preferences over types of SS arrangements

$\Gamma(\tau, \delta)$ does not affect the equilibrium allocation because there is no economic decision

made by the generation 0 old: they simply collect their capital incomes and SS benefits, if

any, in period 1. The preferences of generation 0 would potentially effect the equilibrium

allocation if the SS system were implemented through a voting process, for a SS funding

policy $\delta = 0$ would be their unanimous choice in period 1.[23] Notice also that, in period 1,

an increase in the SS funding policy δ strictly decreases the value of RWB for generation

1 agents.

3.6. Social Security Funding Policy and Intergenerational Equity

Case II: $a_g = 1.0$

Here, we are concerned with another way in which the SSTF could be used"

namely, as redistributive transfers. In this case, the SSTF does not provide SS benefits

because it is not capitalized at all. Instead, the SSTF in each period t is distributed equally

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[23] There arise several interesting issues regarding what would happen if a voting process were introduced. For instance, even in this deterministic economic model, the consequences are not immediately obvious from looking at a time profile of RWB since this only gives the change over time in the lifetime utility of young agents. With a voting process it needs to be determined how the current old agents *feel* about different SS systems, given that they now have only one period of life left.

among all agents alive in period t. Thus, agents do not view the SSTF as an alternative

equivalent means for achieving their desired saving. The type of simulations conducted

for Case I are repeated for Case II. Figures 3.5 and 3.6 summarize the simulation results

obtained for dynamic equilibrium paths, and Table 3.4 displays steady state equilibrium

values of some important variables, including the welfare measures θ and RWB.

Figure 3.5(a) shows again that an increase in the SSTF fraction δ raises the per

capita physical capital stock in each period t and, accordingly, decreases its rate of return.

The reasoning for this crowding-in effect here in Case II is not conceptually different

from the reasoning for the same effect given for Case I; compare Figure 3.3(a). The

increase in the SS funding policy δ decreases the ratio of SS benefits to tax payments in a

strictly monotone fashion. Although government does not save and invest the SSTF as in

Case I, there is still another compensating mechanism of SS tax payments that takes place

outside the SS system. As the government distributes part of the SSTF back to the young

and the remaining part back to the old, all agents experience an increases in their

disposable income.

Since a generation t young agent supplies his labor endowment inelastically to the

production process for wage income in period t, the imposition of a SS tax does not lead

to a distortion in the labor market. Consequently, agents in this model do not distinguish

between different sources of income. In other words, the agents do not care whether their

(a) Physical capital/labor ratio (k)

(b) Net rate of return on physical capital*

(c) Implicit rate of return on SS tax**

* Horizontal axis meets vertical axis at steady state golden rule net rate of return on k, r* = .012.
** Horizontal axis meets vertical axis at the gross population growth rate (1 + n) = 1.012.
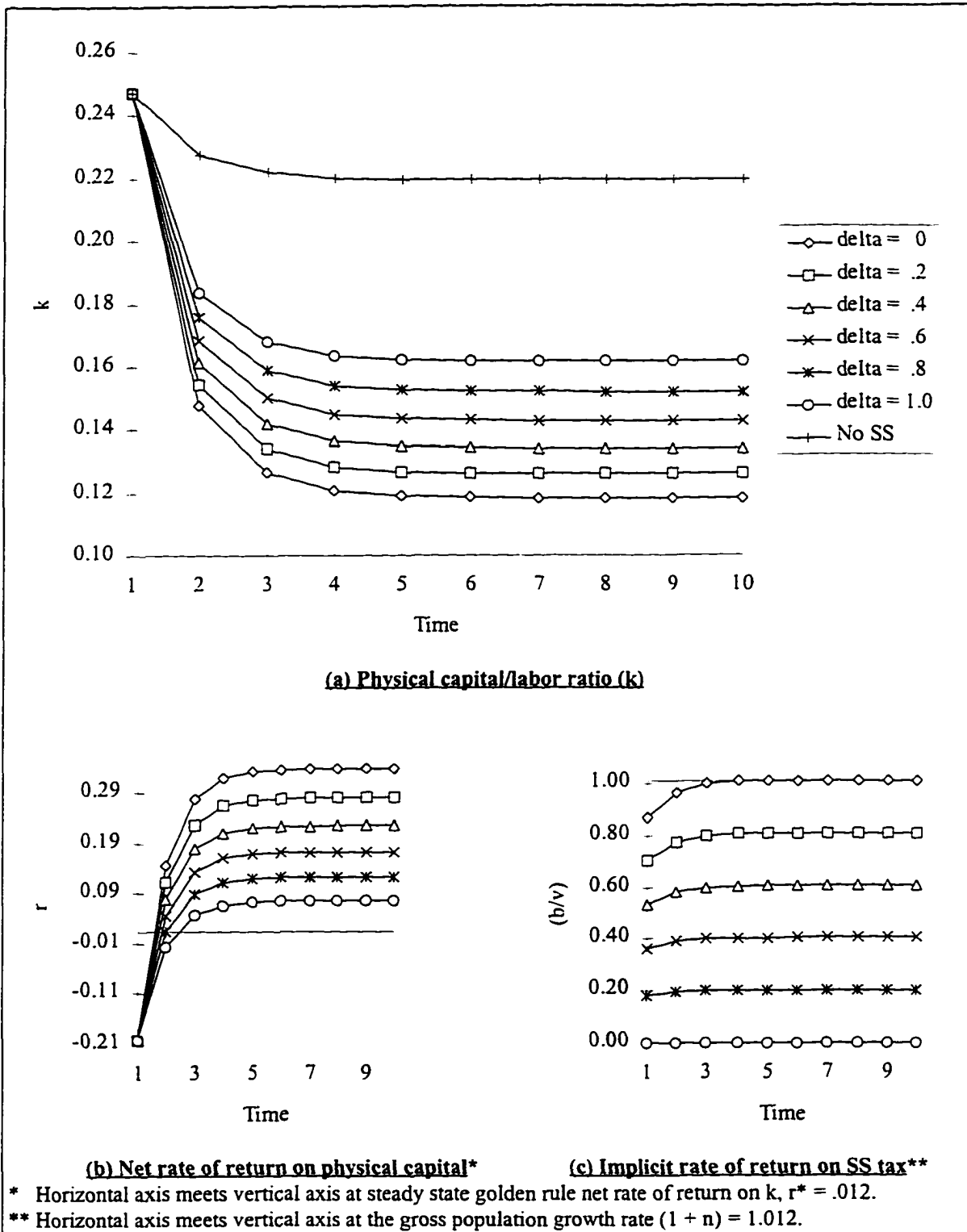
Figure 3.5: Time paths of key variables as the SS funding policy δ changes from 0 to 1:
Case II (a_g = 1.0)
---τ = .2; n = .012, A = 1.0, α = .3, β = .5

(a) Social security actuarial status (θ)*

* For a given δ, SS actuarial status (θ) is constant over time.

(b) Relative welfare gain (RWB)
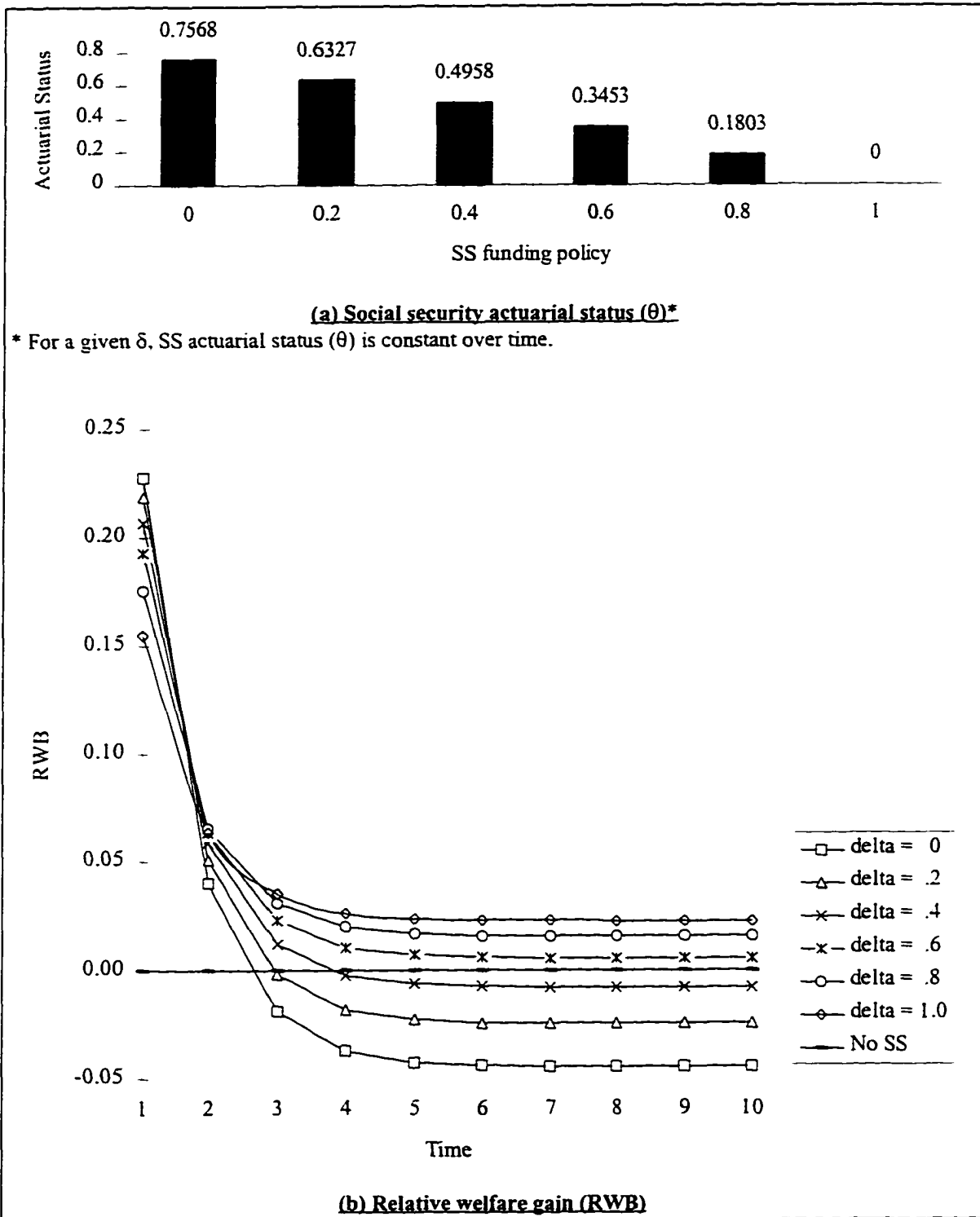
Figure 3.6: Time profiles of intergenerational equity measures as the SS funding policy
δ changes from 0 to 1: Case II ($a_g = 1.0$)
---τ = 0; n = .012, A = 1.0, α = .3, β = 0.5

Table 3.4: Steady state equilibrium and intergenerational equity for the economic model
under alternative SS funding policies δ: Case II ($a_g = 1.0$)
—$\tau = .2$, n = .012, A = 1.0, $\alpha = .3$, $\beta = .5$

Variable	$\Gamma(.2, .0)$	$\Gamma(.2, .2)$	$\Gamma(.2, .4)$	$\Gamma(.2, .6)$	$\Gamma(.2, .8)$	$\Gamma(.2, 1.0)$	$\Gamma(0, 0)$
$\bar{k}$	.1182	.1259	.1340	.1427	.1518	.1615	.2194
$\bar{r}$	.3373	.2797	.2248	.1725	.1226	.0749	-.0133
$\bar{w}$	.3689	.3759	.3831	.3903	.3976	.4051	.4441
$\bar{v}$	.0738	.0752	.0766	.0781	.0795	.0810	0
$\overline{SSTF}^i$	0	.0150	.0306	.0468	.0636	.0810	0
$\bar{T}$	-	.0076	.1564	.0236	.0320	.0408	0
$\bar{b}$	.0747	.0609	.0465	.0316	.0161	0	0
$\bar{s}^p$	.1196	.1274	.1356	.1444	.1536	.1635	.2220
$\bar{c}^y$	.1755	.1809	.1862	.1964	.1965	.2014	.2220
$\bar{c}^o$	.2347	.2315	.2281	.2244	.2206	.2165	.1926
$\bar{y}$	.5270	.5370	.5472	.5576	.5681	.5787	.6344
$\bar{U}$	-1.595	-1.587	-1.579	-1.574	-1.569	-1.566	-1.576
$\bar{U}*$	40.51	41.35	42.05	42.63	44.10	43.25	42.40
$\overline{WB}$	-1.892	-1.054	-.3473	.2259	0.6608	.9517	0
$\bar{b}/\bar{v}$	1.012	.8096	.6072	.4048	.2024	0	-
$\bar{\theta}$	.7568	.6327	.4958	.3453	.1803	0	-
$\overline{RWB}$	-.0446	-.0248	-.0083	.0054	0.0156	.0224	0

income comes from SS benefits or from government redistributive transfers. The fact that

agents do not distinguish between sources of income has an important implication: A

funded SS system with a "government redistributive transfer expenditure only" fiscal

policy could be equivalent to a pay-as-you-go SS system with a lower SS tax. More

precisely, they could have exactly the same effect on the level of per capital physical

capital as long as the following modified effective SS tax rate $\hat{\tau}$ remains unchanged:

$$\hat{\tau} = \tau\left[\frac{1}{n+2}\delta\right] \qquad (3\text{-}4)$$

This is why a higher SSTF fraction δ in Case II has a similar effect to a lower SS tax rate

τ on personal saving.

For this reason, again, the simulation results for the SS funding policy δ in Case II

should not be so different from those for the SS tax policy τ and thus funding policy δ in

Case I, except concerning the SS actuarial status of agents. Figure 3.6(a) shows that

actuarial status varies over different SS funding policies δ in a dramatic fashion. This is

because, as part of SS benefits that would have been realized in Case I are instead

distributed out in the form of government redistributive transfers, an increase in the SS

funding policy δ tends to decrease. pay-as-you-go SS benefits. For instance, when the SS

funding policy δ is 1, there is no SS benefit at all and, thus, the value of actuarial status θ

is simply 0. Notice, however, that compensation takes place *outside* the SS system in the form of transfer payments when young and old.

It is rather straightforward to discuss the intergenerational equity consequences of the SS funding policy δ. As Figure 3.6(b) shows, a higher δ yields a higher value of RWB. The ordering of steady state RWB is given by

$$\overline{RWB}(\delta = 1.0) > \overline{RWB}(\delta = .8) > \overline{RWB}(\delta = .6) > \overline{RWB}(\delta = .4)$$
$$> \overline{RWB}(\delta = .2) > \overline{RWB}(\delta = 0).$$

The ordering can be explained in terms of the steady state rate of return on physical capital. As the gap between the net population growth rate $n = .012$ and the return rate $\bar{r}$ becomes smaller, a higher steady state level $\overline{RWB}$ is achieved. The highest level $\overline{RWB}$ occurs when the net return rate $\bar{r}$ is the closest to $n = .012$, which occurs under the SS funding policy $\delta = 1$. Then, we can ask what specification of the SS funding policy for the given SS tax rate $\tau = .2$ would bring $\bar{r}$ and $n = .012$ into equality. Although it can also be obtained as a matter of calculation from Feature 1 in Section 2.12, Table 3.3 immediately shows that the economy experiences too high a level $\bar{r}$, and hence under-accumulation of physical capital, for all tested SS funding policies δ. In particular, for the given SS tax rate $\tau = .2$, the SS funding policy alone cannot achieve the Pareto optimal golden rule allocation in steady state. This suggests that, in certain cases, SS tax policy is

more effective than SS funding policy in terms of achieving a Pareto optimal SS arrangement.

A potential conflict across generations is again seen in the time profile of RWB in Figure 3.6(b). In period 1, its ordering is exactly reversed from what it is in steady state:

$$RWB_1(\delta = 0) > RWB_1(\delta = .2) > RWB_1(\delta = .4) > RWB_1(\delta = .6)$$

$$> RWB_1(\delta = .8) > RWB_1(\delta = 1.0)$$

As Figure 3.6(b) indicates, there is no SS funding policy that dominates all other funding policies in the sense that it yields a higher RWB for all generations.

As is obvious, the generation 0 old agents in period 1 prefer a lower SS funding policy δ because it implies a higher (modified effective) pay-as-you-go SS tax rate in period 1 and hence higher benefits for them. However, their preferences over the type of SS system do not affect the equilibrium allocation since the have no decisions to make. If the SS system in period 1 were implemented through a voting process, a pay-as-you-go SS system ($\delta = 0$) would be the unanimous choice by all agents in period 1, even though this would lead to the lowest level of steady state RWB.[24]

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[24] It should be noted that, even when the SSTF is not validated into equity claims, there is no financial strain experienced in the present economic model, (i.e., no particular potential drag effect of the government fiscal policy in Case II). This is fundamentally because, although SS benefits are determined partly by how a SS system is arranged and partly by how the SSTF is used by the government fiscal policy, an agent's rationality necessitates to dismantle a possible illusion over the government behavior on the SSTF. Here suppose that the SS benefits structure is internally inconsistent in such a way that each generation t old agent's SS benefits in period t + 1 are also replaced by a fixed fraction of his wage income in period t. Then it is highly probable that the SS system may experience a kind of financial squeeze. For

3.7. Social Security Funding Policy and Intergenerational Equity

Case III: $a_g = a_k = 0$

We have been considering government fiscal policies involving either physical

capital investment or redistributive transfer of the SSTF, and their respective

intergenerational equity consequences from a lifetime utility perspective. Here we deal

with another type of government fiscal policy: The SSTF in period t is assumed to be

expended as human capital investment that augments the production of the raw labor of

generation t young agents. [25] The simulations conducted in Case I and Case II are

repeated with two different specifications for the efficiency parameter λ appearing in

relation (2.33): $\lambda = 1.0$ (low labor productivity) and $\lambda = 3.0$ (high labor productivity).

Figure 3.7 and 3.8 illustrate the simulation results for dynamic equilibrium paths, and

Table 3.5 displays steady state results.

First consider the case $\lambda = 1.0$. Table 3.5 shows that, although government saving

is not invested as physical capital, a higher SS funding policy δ still leads to an increased

steady state level for per capital physical capital $\bar{k}$, implying that a higher δ tends to

---

instance, imagine what would happen when $\delta = 1.0$ in Case II: Simply there is no resource to cover the SS benefits of the generation t old in period t + 1 at all. Moreover, the passage of resolving the potential financial insolvency problem dramatically affect intergenerational equity, implying that, when these elements are ingrained together in the SS benefits structure, a discussion over intergenerational equity consequences of a SS system tends to be inherently complicated.

[25] Notice that, throughout history, some portion of old age security has been met by an institution called "family" in the form of parents' *human capital investment* in their children when young and children's care for them when old. This has been particularly true in periods when old age security has not been socially institutionalized: The first SS system was implemented in Germany a mere 100 years ago. That is, although not included in standard structure of SS systems, human capital investment is one of the traditional ways to secure old age security. See Becker (1987, 1991) for this practice, and see Meyer (1987) for the origin of the SS system.

(a) Wage rate (w) when λ = 1.0

(b) Wage rate (w) when λ = 3.0

(c) Social security actuarial status (θ)
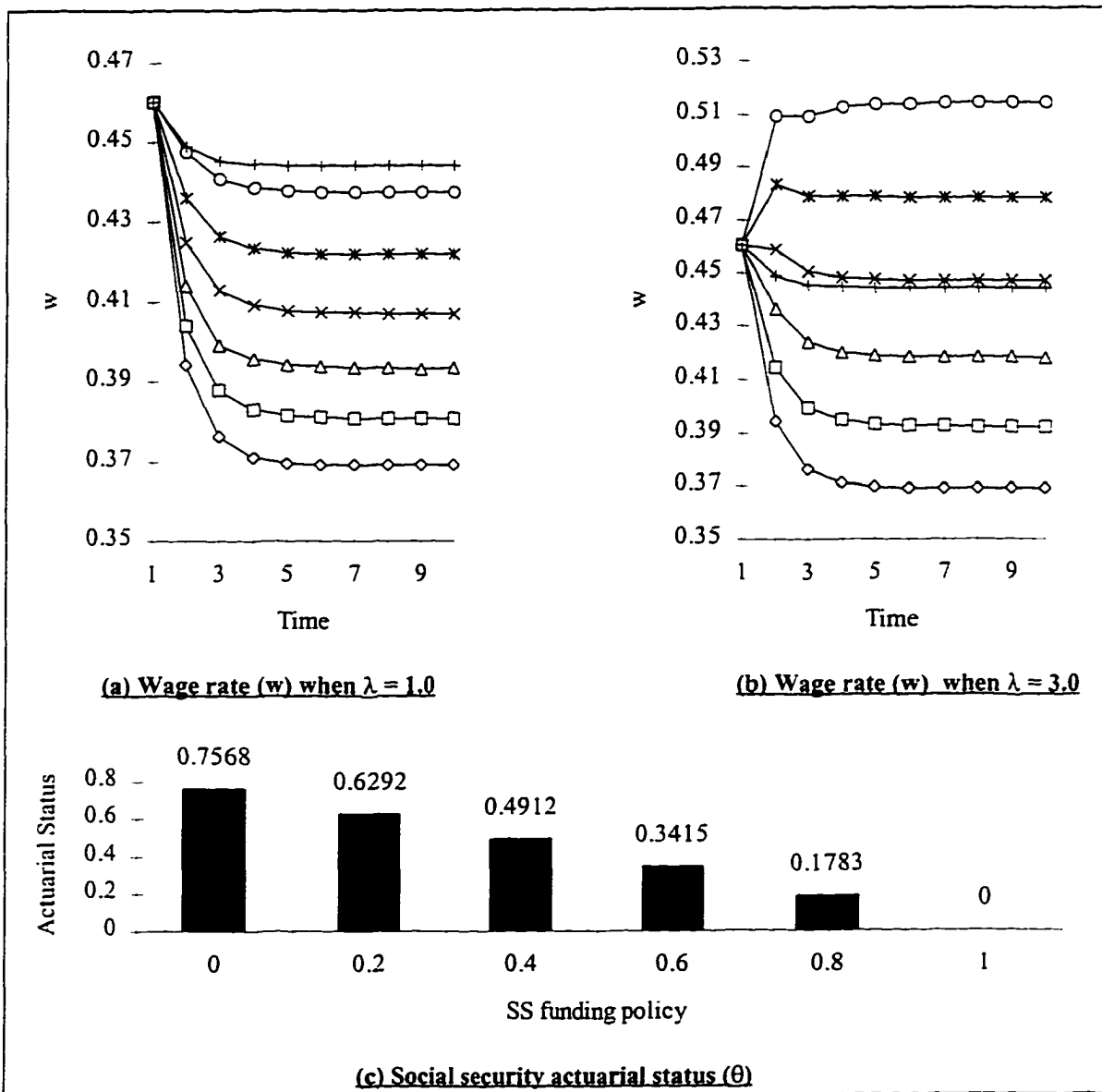
Figure 3.7: Time paths of the wage rate as SS funding policy changes from 0 to 1:
Case III ($a_g = a_k = 0$)
---$\tau = .2$; $n = .012$, $A = 1.0$, $\alpha = .3$, $\beta = .5$, $\lambda = \{1.0, 3.0\}$

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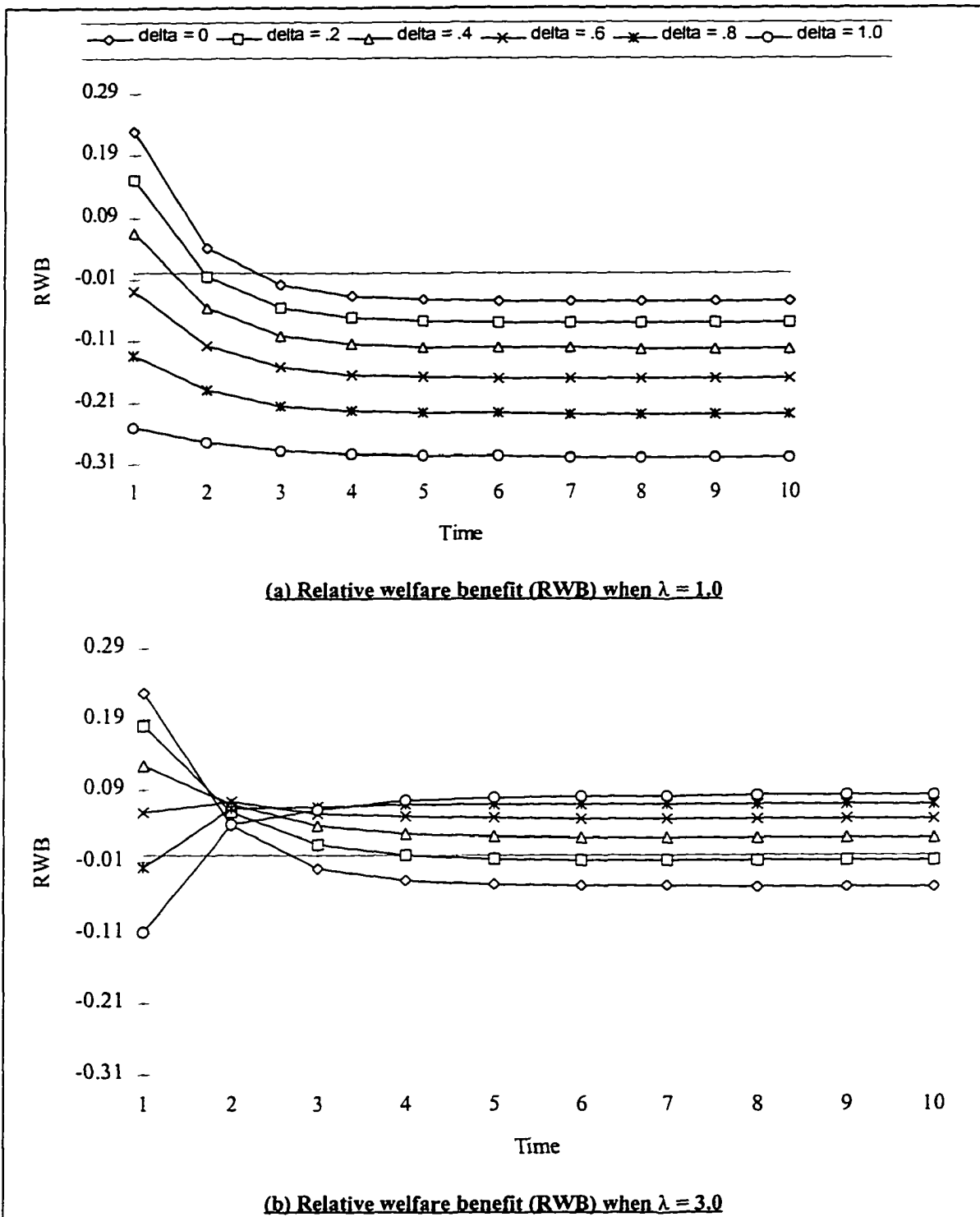Figure 3.8: Time profiles of intergenerational equity measures as the social security
funding policy δ changes from 0 to 1: Case III ($a_g = a_k = 0$)
---$\tau = .2$; $n = .012$, $A = 1.0$, $\alpha = .3$, $\beta = .5$, $\lambda = \{1.\ 3\}$

Table 3.5: Steady state equilibrium and intergenerational equity for the economic model under alternative SS funding policies δ: Case III ($a_g = a_k = 0$) ---τ = .2; n = .012, A = 1.0, α = .3, β = .5, $\lambda \in \{1.0, 3.0\}$

Variable		$\Gamma(.2, .0)$	$\Gamma(.2, .2)$	$\Gamma(.2, .4)$	$\Gamma(.2, .6)$	$\Gamma(.2, .8)$	$\Gamma(.2, 1.0)$
$\bar{k}$		.1182	.1249	.1323	.1404	.1495	.1595
$\bar{r}$		.2867	.2797	.2361	.1855	.1349	.08429
$\bar{\bar{w}}$		.3689	.3751	.3816	.3885	.3958	.4036
$\bar{e}$	λ =1.0	1	1.015	1.031	1.048	1.065	1.084
	λ =3.0	1	1.046	1.096	1.150	1.209	1.274
$\bar{w}$	λ =1.0	.3689	.3807	.3934	.4070	.4217	.4375
	λ =3.0	.3689	.3903	.4181	.4468	.4786	.5142
$\bar{v}$	λ =1.0	.0738	.0761	.0787	.0814	.0843	.0875
	λ =3.0	.0738	.0785	.0836	.0894	.0957	.1028
$\bar{S}\bar{S}\bar{T}F^i$	λ =1.0	0	.0152	.0315	.0488	.0675	.0875
	λ =3.0	0	.0157	.0335	.0536	.0766	.1028
$\bar{b}$	λ =1.0	.0747	.0617	.0478	.0330	.0171	0
	λ =3.0	.0747	.0635	.0508	.0362	.0194	0
$\bar{s}^p$	λ =1.0	.1196	.1283	.1380	.1489	.1611	.1750
	λ =3.0	.1196	.1322	.1467	.1364	.1829	.2057
$\bar{c}^y$	λ =1.0	.1755	.1762	.1767	.1767	.1762	.1750
	λ =3.0	.1755	.1816	.1878	.1940	.2000	.2057
$\bar{c}^o$	λ =1.0	.2347	2268	.2184	.2095	.1999	.1898
	λ =3.0	.2347	.2357	.2321	.2299	.2269	.2230
$\bar{y}$		.5270	.5358	.5451	.5549	.5654	.5766
$\bar{U}$	λ =1.0	-1.595	-1.610	-1.627	-1.648	-1.673	-1.702
	λ =3.0	-1.595	-1.580	-1.566	-1.555	-1.546	-1.541
$\bar{U}*$	λ =1.0	40.51	39.01	37.25	35.17	32.70	29.75
	λ =3.0	40.51	42.02	43.36	44.50	45.37	45.90
$\overline{WB}$	λ =1.0	-1.892	-3.385	-5.145	-7.227	-9.697	-12.65
	λ =3.0	-1.892	-.3845	.9597	2.097	2.968	3.497
$\bar{b}/\bar{v}$		1.012	.8096	.6072	.4048	.2024	0
$\bar{\theta}$		.7568	.6292	.4912	.3415	.1783	0
$\overline{RWB}$	λ =1.0	-.0446	-.0800	-.1215	-.1705	-.2288	-.2981
	λ =3.0	-.0446	-.0090	.0226	.0495	.0701	.0826

require agents to self-finance their old age consumptions to a greater extent. Notice that their young age income increases due to the enhanced labor skill but there is no direct compensating mechanism for the SS taxes paid when young either inside (Case I) or outside (Case II) the SS system.[26]

Immediately apparent from Figure 3.7(a) is that the wage rate decreases over time in a strictly monotone fashion in the presence of the increase in labor efficiency. Figure 3.7(c) also illustrates the time profiles of SS actuarial status θ, in which the value of θ, being constant over time, decreases with higher δ for the same reason as discussed in Section 3.6.

From Figure 3.8(a) it is obvious that a higher SS funding policy δ uniformly reduces relative welfare benefit RWB_t for all generations t when $\lambda = 1$. The ordering of steady state RWB is given by

$$0 > \overline{RWB}(\delta = 0) > \overline{RWB}(\delta = .2) > \overline{RWB}(\delta = .4) > \overline{RWB}(\delta = .6)$$
$$> \overline{RWB}(\delta = .8) > \overline{RWB}(\delta = 1.0).$$

---

[26] It is tempting to think that this result from a lower effective tax rate, modified or not. However, this reasoning is not applicable to the current case because the increase in δ implies an increase in human capital investment, which increases efficiency e_t and hence the effective labor force $L_t = e_t N_t$, and which in turn has a depressing effect on the steady state per-capita physical capital. Instead, a detailed explanation can be found from a comparison of the incremental saving and efficiency with respect to the change in δ. For instance, the sign of $d\bar{k}/d\delta$ depends on $(\bar{e}\,d\bar{s}^P/d\delta - \bar{s}^P\,d\bar{e}/d\delta)$. Noting that paths associated with different SS funding policies δ do not intersect, the ordering attained in the ultimate steady state is the same as the ordering in each period t. Hence, higher values of δ are associated with higher levels of per-capita physical capital in non-steady state as well.

Thus, for the given SS tax policy $\tau = .2$, the highest value of steady state RWB occurs when the SS funding policy is $\delta = 0$, and this value is negative. Note that the ordering is reversed from that of Case II. This result shows that, for generation t agents, the beneficial effects of increased wage rates in period t and t + 1 due to an increase in δ, and hence an increase in human capital investment, is offset by the negative effects of the decrease in SS benefits proportional to the SSTF and of the decrease in capital income. These negative effects are most dramatic for generation 1 agents. For example, given an increase in δ to 1, there are no beneficial effects for generation 1 agents, only negative effects coming from a decrease in capital income and no SS benefits. If the choice of a SS system were decided in period 1 through a voting process, a pay-as-you-go SS system ($\delta = 0$) would again be the unanimous choice of all period 1 agents.

Now consider the case when the efficiency parameter is given by $\lambda = 3.0$, implying that human capital investment is more productive. As Table 3.5 indicates, the steady state level $\bar{k}$ of per capita physical capital is the same for different specifications of the efficiency parameter λ. The reason for this can easily be easily inferred from the basic causal difference equation (2-48) for economic model at hand. [Note, however, that this is not applicable to the non-steady state.] Given that $\bar{k}$ is invariant to changes in λ, the same must be true for the return rate $\bar{r}$ and the effective wage rate $\bar{\bar{w}}$. However, the actual wage rate $\bar{w}$ faced by agents is sensitive the changes in λ because more productive

human capital investment leads to a higher steady state efficiency level $\bar{e}$ and, consequently, a higher steady state wage rate.

Time paths for the wage rate w_t given $\lambda = 3.0$ under alternative SS funding policies $\delta = 3.0$ are depicted in Figure 3.7(b). These time paths are not always monotone, in contrast to the findings for $\lambda = 1.0$. That is, even in the presence of monotone decrease in per-capita physical capital, the time path for the wage rate is not necessarily monotone. This is because higher value for the efficiency parameter leads to increased wage rate over time in such a way that efficiency increase dominate the effective wage rate decrease for some SS funding policies especially in the initial periods.

Moreover, as indicated in Table 3.5, the steady state implicit rate is the same for $\lambda = 1.0$ and $\lambda = 3.0$. This is because, in steady state, the implicit rate of return is not a function of λ. The same is true for SS actuarial status. These invariance findings show how the equilibrium wage rate and rate of return on physical capital respond to changes in the SS system in a complicated way.

Table 3.5 also shows that the steady state RWB levels for $\lambda = 3.0$ are quite different from the corresponding levels for $\lambda = 1.0$. First, the ordering is completely reversed. For example, the highest steady state RWB level occurs for $\lambda = 3.0$ when the SS system is fully funded ($\delta = 1.0$). The complete ordering for $\lambda = 3$ is as follows;

$$\overline{RWB}(\delta = 0) > \overline{RWB}(\delta = .2) > \overline{RWB}(\delta = .4) > \overline{RWB}(\delta = .6)$$
$$> \overline{RWB}(\delta = .8) > \overline{RWB}(\delta = 1.0).$$

This reversed ordering shows that, in steady state, when human capital investment is more productive, i.e., when $\lambda = 3.0$, the beneficial effects of an increased steady state wage rate offset the negative effects of a decrease in SS benefits (proportional to the SSTF fraction) and capital income (due to a lower return rate).

This reasoning, however, is not applicable to period 1 and period 2. For instance, as seen in Figure 3.8(b), in period 1 the RWB orderings for $\lambda = 1.0$ and $\lambda = 3.0$ are the same. This can be accounted for by the fact that generation 1 young agents do not enjoy any beneficial wage rate effects. Note that government human capital investment begins to take place in period 2 and after. The relatively higher value of RWB for generation 1 agents under $\lambda = 3.0$ stems from the higher capital income received in period 2 due to the relatively more abundant effective labor force and the resulting increase in the rate of return on physical capital in period 2.

As discussed in Footnote 23, Section 3.7, a maintained SS benefits structure is highly likely to lead to financial strain when the SSTF is not in the form of an equity claim on real assets that can provide future SS benefits. Moreover, it is often believed that, when the financial squeeze is resolved in the form of an increased SS tax rate in future periods, there is be a shift of (financial) burden to future generations. However, the simulation findings when the productivity of human capital investment is high ($\lambda = 3.0$) suggest that these beliefs may not always be warranted. Instead, the ordering of RWB when $\lambda = 3.0$ provide an example where human capital investment is productive enough

to enhance wage rates and lifetime utilities of future generations, implying that an increased SS tax rate in future periods would not impose an onerous burden on the future generations. However, the simulation findings when the productivity of human capital investment is low ($\lambda = 1.0$) caution that it would be a mistake to assume that future generations do not bear a financial burden simply because the SSTF is used for their sake.

Overall, the time paths for RWB are dramatically different for varying values of the efficiency parameter λ, implying that the productivity of human capital investment is critically important for determining the intergenerational equity effects of alternative SS systems. In particular, this finding suggests that the intergenerational equity consequences of government fiscal policy regarding the SSTF depend not only on who will receive the benefits of the policy but also on the extent of the benefits. That is, as shown in a consistent way for all simulations conducted so far, the performance of the SSTF critically matters, in addition to what government does with the SSTF.

3.8. Sensitivity Tests

Simulations are run for a given government fiscal policy-mix use of the SSTF (Case IV: $a_g = .5$ and $a_k = 1.0$). We first run simulations for baseline values for the parameters, as set out in Chapter 2 and maintained throughout Chapter 3. Then, these simulations are rerun assuming different values for the net population growth rate, n, the share of labor in the production function, $(1 - \alpha)$, and the consumption time preference in the utility function, β, respectively. By doing so, we can investigate how the relationship

between the SSTF and intergenerational equity depends on structural aspects such as a decreased population growth rate [from n = .012 to n = -.012], a lower share of labor [from $(1 - \alpha) = .7$ to $(1 - \alpha) = .65$], and a higher time preference for young age consumption [from $\beta = .5$ to $\beta = .55$].

Table 3.6 and Figure 3.9 show the sensitivity of the two steady state intergenerational equity measures, actuarial status $\bar{\theta}$ and relative welfare benefit $\overline{RWB}$, to a decrease in the net population growth rate n. For both the baseline case (n = .012) and the case with n = -.012, these measures exhibit the same ordering over different SS funding policies δ. These orderings are as follows:

$$1 > \bar{\theta}(\delta = 0) > \bar{\theta}(\delta = .2) > \bar{\theta}(\delta = .4) > \bar{\theta}(\delta = .6)$$

$$> \bar{\theta}(\delta = .8) > \bar{\theta}(\delta = 1.0);$$

(3-5)

$$\overline{RWB}(\delta = .8) > \overline{RWB}(\delta = 1) > \overline{RWB}(\delta = .6) > \overline{RWB}(\delta = .4)$$

$$> \overline{RWB}(\delta = .2) > \overline{RWB}(\delta = 0).$$

(3-6)

A higher SS funding policy δ monotonically decreases the steady state SS actuarial status $\bar{\theta}$ and thus the SS actuarial status in every period t since it is time

Table 3.6: Sensitivity tests with respect to n, α, and β for Case IV ($a_g = .5$, $a_k = 1.0$)
—τ = .2; n = {.012, -.012}, α = {.3, .35} β = {.5, .55}

	$\Gamma(0, 0)$	$\Gamma(.2, 0)$	$\Gamma(.2, .2)$	$\Gamma(.2, .4)$	$\Gamma(.2, .6)$	$\Gamma(.2, .8)$	$\Gamma(.2, 1.0)$
$\bar{k}$							
Baseline	.2194	.1182	.1299	.1426	.1564	.1716	.1882
n = -.012	.2271	.1224	.1343	.1474	.1617	.1773	.1944
α = .35	.1742	.0951	.1044	.1145	.1255	.1273	.1503
β = .55	.1888	.0990	.1091	.1202	.1324	.1458	.1607
$\bar{r}$							
Baseline	-.1326	.3373	.2522	.1731	.0993	.0303	-.0343
n = -.012	-.1531	.3056	.2280	.1460	.0741	.00693	-.0560
α = .35	-.0898	.6152	.5201	.4316	.3491	.2720	.1997
β = .55	-.3619	.5140	.4143	.3217	.2353	.1545	-.0789
$\bar{s}^p$							
Baseline	.2220	.1196	.1238	.1287	.1342	.1406	.1481
n = -.012	.2243	.1209	.1251	.1299	.1354	.1419	.1493
α = .35	.1763	.0962	.0998	.1037	.1081	.1130	.1186
β = .55	.1910	.1002	.1032	.1068	.1111	.1162	.1221
$\bar{s}^g$							
Baseline	0	0	.00759	.0156	.0241	.0330	.0424
n = -.012	0	0	.0077	.0158	.0243	.0333	.0428
α = .35	0	0	.0059	.0127	.0189	.0260	.0335
β = .55	0	0	.0072	.0148	.0229	.0314	.0405
$\bar{s}^n$							
Baseline	.2220	.1196	.1314	.1443	.1583	.1737	.1905
n = -.012	.2243	.1209	.1327	.1457	.1598	.1725	.1921
α = .35	.1763	.0962	.1057	.1159	.1270	.1390	.1521
β = .55	.1910	.1002	.1104	.1217	.1340	.1476	.1625
$\bar{U}*$							
Baseline	42.40	40.51	41.71	42.62	43.22	43.49	43.41
n = -.012	42.23	40.34	41.54	42.44	43.04	43.32	43.24
α = .4	30.75	21.47	23.62	25.51	27.14	28.51	29.60
β = .55	43.84	37.95	39.71	41.19	42.37	43.24	43.77
$\bar{b}/\bar{v}$							
Baseline	-	-1.0120	.9348	.8418	.7346	.6145	.4829
n = -.012	-	.9880	.9127	.8220	.7174	.6004	.4720
α = .35	-	1.0120	.9816	.8935	.8095	.7112	.5999
β = .55	-	1.0120	.9510	.8715	.7564	.6642	.5395

Table 3.6: Continued

	$\Gamma(0, 0)$	$\Gamma(.2, 0)$	$\Gamma(.2, .2)$	$\Gamma(.2, .4)$	$\Gamma(.2, .6)$	$\Gamma(.2, .8)$	$\Gamma(.2, 1.0)$
$\overline{WB}$							
Baseline	0	-1.892	-.6885	.2199	.8194	1.091	1.008
n = -.012	0	-1.893	-.6942	.2125	.8127	1.087	1.012
α = .35	0	-9.278	-7.130	-5.237	-3.605	-2.240	-1.152
β = .55	0	-5.893	-4.132	-2.655	-1.472	-.6017	-.0660
$\overline{\theta}$							
Baseline	0	.7568	.7465	.7176	.6682	.5964	.5000
n = -.012	0	.7568	.7464	.7173	.6680	.5962	.5000
α = .35	0	.6265	.6326	.6241	.6000	.5753	.5000
β = .55	0	.6684	.6724	.6594	.6277	.5753	.5000
$\overline{RWB}$							
Baseline	0	-.0446	-.0162	.0059	.0193	.0257	.0238
n = -.012	0	-.0448	.0164	.0050	.0193	.0258	.0240
α = .35	0	-.3017	-.2319	-.1703	-.1172	-.0729	-.0375
β = .55	0	-.1344	-.0943	-.0606	-.0336	-.0137	-.0015

(a) Social security actuarial status
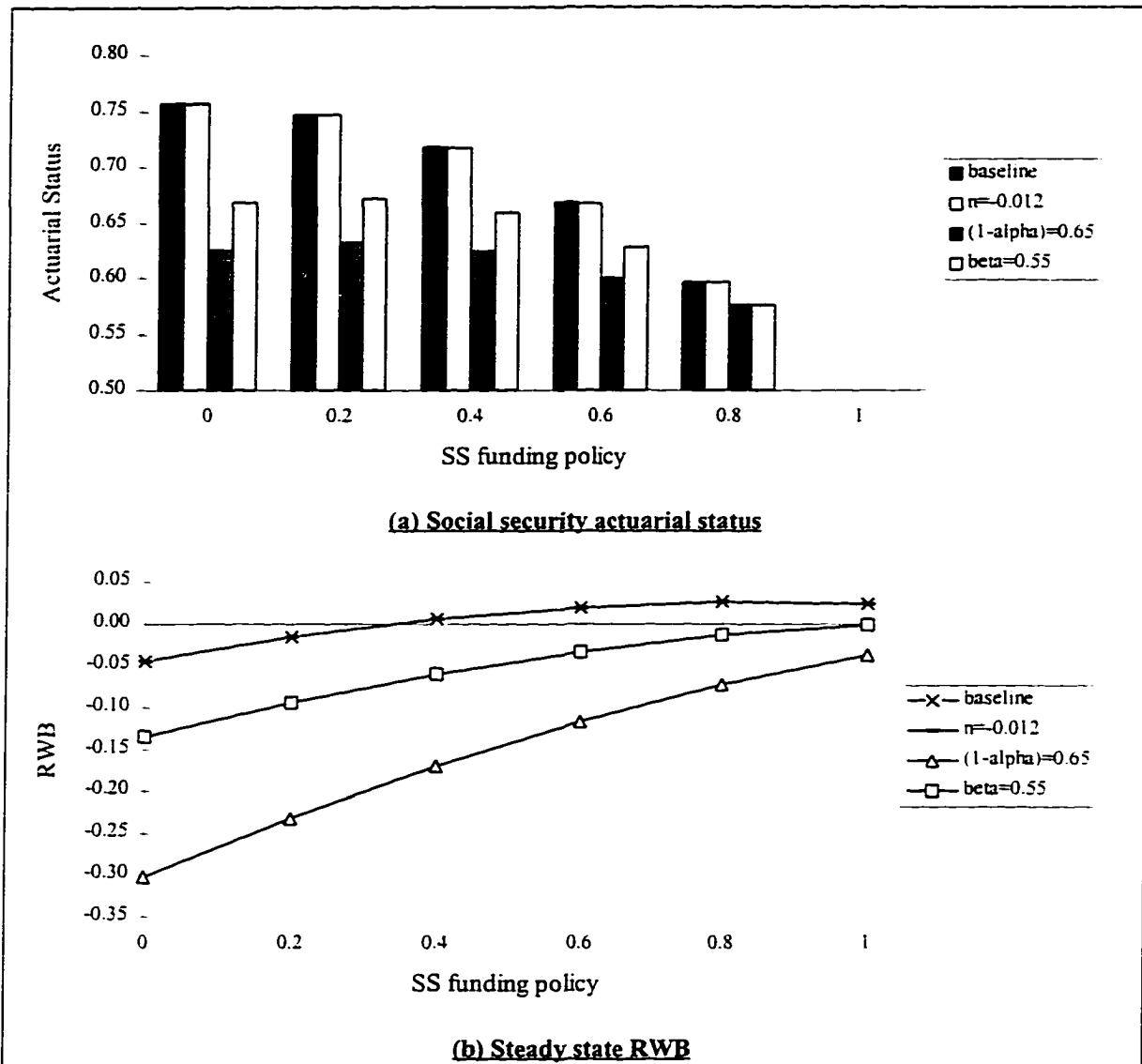

(b) Steady state RWB

Figure 3.9: Sensitivity of intergenerational equity measures to changes in n, α, and β for Case IV (a_g = .5, a_k = 1.0)
--τ = .2; n = {.012, -.012}, (1 - α)={.7, .65}, β={.5, .55}

invariant. As usual, a greater amount of SS tax payments are distributed out in terms of transfers with a higher δ. The ordering of $\overline{RWB}$ shows that the case at hand is intermediate between Case I and Case II. That is, in Case I, the highest $\overline{RWB}$ occurs when $\delta = .6$, in Case II it occurs when $\delta = 1.0$, and here it occurs when $\delta = 8$. More importantly, the fact that the same ordering of $\bar{\theta}$ and $\overline{RWB}$ is obtained over these two distinctively different demographic states suggests that these intergenerational equity measures are not sensitive to changes in the (net) population growth rate n.

Although it is difficult to conduct comparative dynamic analyses in most cases, we are fortunate that we can analyze the effects of changes in the population growth rate on the optimal steady SS funding policy δ^*. From Feature 1 in Section 2.12, we can easily find that the sign of $(\partial \delta^* / \partial n)$ is opposite to the sign of $(\partial M / \partial n)$, where M is defined by (2-68). Thus, the *partial* derivative of M with respect to n will show, for given τ, a_g, and a_k, the impact of a change in n on δ^*:

$$\frac{\partial M}{\partial n}\bigg|_{\tau, a_g, a_k} = \frac{\partial \left[(1 - \beta)\left[\frac{n + 1}{n + 2}a_g + (1 - a_g)a_k \right] + \beta \left[1 - \frac{1}{n + 2}a_g \right] \right]}{\partial n} \Bigg|_{\tau, a_g, a_k}$$

$$= (1 - \beta)\frac{1}{(n + 2)^2}a_g - \beta \frac{1}{(n + 2)^2}a_g$$

$$= \frac{(1 - 2\beta)}{(n + 2)^2}a_g. \tag{3-7}$$

The sign of $(\partial M / \partial n)$ depends on the value of the consumption time preference β. It is negative when $\beta > .5$ and positive when $\beta < .5$. So, when $\beta > .5$, a decrease in the net population growth rate n decreases the optimal SS funding policy δ^*, and when $\beta < .5$, it increases δ^*. When $\beta = .5$, as is in our baseline specification of the parameter, there is no impact of the population growth rate on the optimal SS funding policy at all, explaining why we obtained the same ordering of $\overline{RWB}$ in the previously discussed simulations. Notice that δ^* is also invariant to the changes in n when $a_g = 0$ (Case I).

As indicated in Table 3.6, for a given SS funding policy δ, a lower value of n tends to decrease steady state intergenerational equity measured by SS actuarial status. Although its impact on $\overline{RWB}$ is not so uniform, the fact that the ordering (3-6) remains unchanged as n decreases indicates that changes in n have a weak impact on $\overline{RWB}$. Indeed, Miguel-Angel and Lopez-Garcia (1991), in their study of the role of the population size in a pay-as-you-go SS system, show that the relationship between the population growth rate and lifetime utility is *far from clear*.

This result is in sharp contrast with the general notion that different population growth rates result in a SS system having significantly different economic and welfare consequences. For instance, the popularity of the pay-as-you-go SS system is often accounted for by a positive population growth rate. Since the SS benefits of the generation t old agents in period t + 1 under a pure pay-as-you-go SS system rely on number of generation t + 1 young agents and their work performance, a larger young age

population would seem to imply greater SS benefits for the generation t old agents (positive growth dividend effect).

Yet, this is not the case with our economic model. Given our assumptions on the production technology, a decreased population growth rate increases per capital physical capital in the steady state.[27] Hence, the steady state wage rate also increases, which in turn increases the steady state SS benefits, implying that this wage congestion effect weakens at least partially offsets the negative growth dividend effect on SS benefits due to the decrease in the population growth rate. In general, then, the complex relationship between a SS system and the general equilibrium effects associated with a demographic change need to be carefully investigated before it can be said with certainty that a particular demographic structure makes one SS system preferable to other SS systems.

Table 3.6 and Figure 3.9 also show the sensitivity of the steady state intergenerational equity measures $\bar{\theta}$ and $\overline{RWB}$ to changes in the labor share parameter $(1 - \alpha)$. The simulation results show that $\overline{RWB}$ is highly sensitive to changes in $(1 - \alpha)$ on two counts. First, for two different values of $(1 - \alpha)$, the baseline value $(1 - \alpha) = .7$ and a lower value $(1 - \alpha) = .65$, the orderings of $\overline{RWB}$ over different SS funding policies δ are slightly different. Unlike in the simulation results with the baseline specification $(1 - \alpha) = .7$, the highest $\overline{RWB}$ for $(1 - \alpha) = .65$ occurs when $\delta = 1.0$. For both $(1 - \alpha)$ values, however, we observe that the value of $\overline{RWB}$ decreases as δ increases. Second, and more

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[27] This can be verified from the comparative dynamic study of the change in per capita physical capital with respect to changes in n using (2-69) The sign is unambiguously positive.

importantly, when the share of labor decreases from $(1 - \alpha) = .7$ to $(1 - \alpha) = .65$, agents experience a loss in intergenerational equity as measured by $\overline{RWB}$ for each of the tested SS funding policies δ.

Again, a comparative dynamic analysis of the change in the optimal SS funding policy δ^* with respect to a changes in α provides an explanation for this different ordering of $\overline{RWB}$. More precisely, from condition (2-69) characterizing the steady state golden rule allocation, the derivative of δ^* with respect to α for given τ, a_g, and a_k is as follows:

$$\frac{\partial \delta^*}{\partial \alpha}\bigg|_{\tau, a_g, a_k} = \frac{\partial \left[\dfrac{\tau - (1 - \beta) + \alpha(1 - \alpha)}{M\tau}\right]}{\partial \alpha}$$

$$= \frac{1}{M\tau(1 - \alpha)^2} > 0, \tag{3-8}$$

where $M > 0$ is given by (2-68).

Thus, for given τ, a_g, and a_k, a lower labor share $(1 - \alpha)$ raises the optimal steady state SS funding policy δ^*, which explains the different orderings determined for $\overline{RWB}$ in the simulations. The negative values of $\overline{RWB}$ in Figure 3.9(b) for $(1 - \alpha) = .65$ can be explained again by condition (2-67) characterizing the steady state golden rule allocation. That is, the optimal pay-as-you-go SS rate τ^*, given $(1 - \alpha) = .65$, is negative, meaning

that agents are better off in the absence of a SS system ($\tau = 0$) than in the presence of a

SS system ($\tau > 0$).

Finally, Table 3.6 and Figure 3.9 also show the sensitivity of $\overline{\theta}$ and $\overline{RWB}$ to an

increase in the consumption time preference β from the baseline value .5 to .55. It is

already suggested, from the sensitivity of intergenerational measures to changes in the

population growth rate, that β plays a central role in the determination of the impact of n

on $\overline{RWB}$. These simulation findings exhibit the same qualitative pattern as the

simulation findings for the sensitivity of $\overline{\theta}$ and $\overline{RWB}$ to a decrease in $(1 - \alpha)$, except

with regard to magnitudes. That is, the orderings of $\overline{RWB}$ are different for the different β

values; when the consumption time preference increases from $\beta = .5$ to $\beta = .55$, agents

experience a decrease in steady state θ and RWB for each of the tested SS funding

policies δ. Again, we conclude that intergenerational equity as measured either by θ or by

RWB is sensitive to changes in the consumption time preference parameter, as verified

also in many other studies of SS systems. For instance, Imrohoroglu, Imrohoroglu, and

Joines (1992) show that the optimal SS tax rate is zero when $\beta > .5$ and positive when

$\beta < .5$.

Overall, the population growth rate n appears to have only a weak impact on the

relationship between the SS funding policy δ and intergenerational equity, especially as

measured by $\overline{RWB}$. In contrast, the share of labor $(1 - \alpha)$ and the consumption time

preference β seem to have crucial impacts on this relationship. One implication: A switch

to a more fully funded SS system is justified in response either to a decreased share of labor or an increased consumption time preference for young age consumption, but not necessarily in response to a decrease in the population growth rate.

CHAPTER 4. SUMMARY

4.1. The Economic Model

Understanding how different social security (SS) systems affect the economy and intergenerational equity draws considerable attention, especially in the context of changing structural aspects in the economy. In this dissertation we pay particular attention to the degree to which SS systems are funded and to the alternative government fiscal policies regarding the use of the SSTF in order to investigate the resulting economic and intergenerational equity consequences. To carry out this investigation, we develop a computational two periods lived overlapping generations model in which a wide range of possible SS arrangements and possible government fiscal uses of the SSTF are incorporated in parameterized form: namely, two SS arrangement parameters and two government fiscal policy parameters. This parameterization of the SS system permits the comparative dynamic study of a family of SS systems.

The evolution of the economic model is fully accounted for by a basic causal difference equation (BCDE), equation (2-48) by which the time path of the physical capital/effective labor ratio is traced down. Moreover, the economic model is characterized in terms of three features. Feature 1 provides a steady state golden rule Pareto optimal condition. Feature 2 compares private saving S^p with the SSTF and/or the government saving S^g concerning potential economic effects. Feature 3 reveals that,

under a certain government fiscal policy use of the SSTF, the type of SS arrangement can be irrelevant to the economy.

4.2. Responses of the Economy

The simulation results for SS tax policies τ show that an increase in the tax rate under pay-as-you-go SS system ($\delta = 0$) crowds out per capita physical capital in each time period. This can also be seen analytically. First, the sign of $d\bar{k}/d\tau$ is unambiguously negative. Second, the time paths for k_t generated by the BCDE (2-48) for different τ values do not intersect each other. Putting these two findings together suffices to show why SS tax rate unambiguously crowds out the physical capital/labor ratio in each period t in the sense that $k_t(\tau'') < k_t(\tau')$ if $\tau'' > \tau'$.

The simulation results of SS funding policies δ (Cases I, II, and III) show that, regardless of the type of government fiscal policy use of the SSTF, an increase in the SSTF fraction δ increases the physical capital/(effective) labor ratio in each period. The extent of the crowding-in and its underlying causes are quite different, though. The responses of the economy to changes in the SS funding policy δ under alternative government fiscal use of the SSTF are summarized in Table 4.1.

When the SSTF is used by government purely for physical capital investment expenditure (Case I), the crowding-in of the physical capital/labor ratio due to an increase of the SSTF fraction δ is explained in an intuitively obvious way. In Case I, an increase in

Table 4.1: Effects on key variables and intergenerational equity of alternative SS funding policies δ and alternative government fiscal policy uses of the SSTF
—τ = .2; n = .012, A = 1.0, α = .3, β = .5, λ ∈ {1.0, 3.0}

		Γ(.2, .0)	Γ(.2, .2)	Γ(.2, .4)	Γ(.2, .6)	Γ(.2, .8)	Γ(.2, 1.0)
$\bar{k}$							
Case I		.1182	.1339	.1516	.1715	.1939	.2194
Case II		.1182	.1259	.1340	.1427	.1518	.1615
Case III		.1182	.1249	.1323	.1404	.1495	.1595
$\bar{s}^p$							
Case I		.1196	.1355	.1534	.1735	.1762	.2220
Case II		.1196	.1274	.1356	.1444	.1536	.1635
Case III	λ =1.0	.1196	.1283	.1380	.1489	.1611	.1750
	λ =3.0	.1196	.1322	.1467	.1364	.1829	.2057
$\bar{s}^n$							
Case I		.1196	.1355	.1534	.1735	.1622	.2220
Case II		.1196	.1274	.1356	.1444	.1536	.1635
Case III	λ = 1.0	.1196	.14353	.16947	.1977	.22857	.2625
	λ = 3.0	.1196	.14789	.18015	.19001	.25948	.3085
$\bar{U}*$							
Case I		40.51	42.05	43.05	43.49	43.30	42.40
Case II		40.51	41.35	42.05	42.63		43.25
Case III	λ = 1.0	40.51	39.01	37.25	35.17	32.70	29.75
	λ = 3.0	40.51	42.02	43.36	44.50	45.37	45.90
$\overline{WB}$							
Case I		-1.892	-.3549	.6521	1.090	0.9004	0
Case II		-1.892	-1.054	-.3473	.2258	0.6608	.9517
Case III	λ = 1.0	-1.892	-3.385	-5.145	-7.227	-9.6970	-12.65
	λ = 3.0	-1.892	-.3804	.9597	2.097	2.968	3.497
$\bar{\theta}$							
Case I		.7568	.8670	.9404	0.9927	1.014	1.000
Case II		.7568	.6327	.4958	.3453	.1803	0
Case III	λ	.7568	.6292	.4912	.3415	.1783	0
$\overline{RWB}$							
Case I		-.04462	-.00837	.01538	.02570	.02124	0
Case II		-.04462	-.02487	-.00819	.00533	.01558	.02245
Case III	λ = 1.0	-.04462	-.07984	-.1214	-.1704	-.22880	-.2982
	λ = 3.0	-.04462	-.00907	.02263	.04945	.07000	.08248

δ is equivalent to a decrease in τ as long as the effective tax rate $\tilde{\tau}$ remains unchanged. Hence, the crowding-in associated with the SSTF in Case I is explained by the crowding out phenomenon associated with changes in τ under a pure pay-as-you-go SS system. For the same reason, the exact magnitude of δ in Case I is irrelevant to the economy, and the same time path for per-capita physical capital can be generated by two distinctively different SS systems, such as a SS system with low τ and low δ and a SS system with high τ and high δ.

The crowding-in process in Case II is not so different from the one in Case I. (Recall the modified effective tax rate.) The only two differences lie, first, in the compensating mechanism of SS taxes paid, and, second, in the extent of the crowding-in. This is because, first, part of compensation of SS taxes paid by generation t young agents takes place outside the SS system in terms of redistributive transfers and, second, part of the SSTF is transferred to young age agents. One implication: Agents' rationality alone prevents a possible illusion regarding the government fiscal use of the SSTF so that there is no fiscal drag effect even when the SSTF is not transferred into an equity claim

Contrary to an original conjecture that higher labor efficiency due to government human capital investment on labor would depress the physical capital/effective labor ratio, an increase in the SSTF fraction δ still raises the physical capital/labor ratio in Case III. This is explained by the fact that the enhanced labor skills of agents increases their wage incomes and, in the absence of any direct compensating mechanism by government, private agents tend to self-finance their old age consumption to a relatively greater extent.

4.3. Actuarial Status as a Measure of Intergenerational Equity

In the simulations of alternative SS tax rates τ under a pure pay-as-you-go SS system, the values of SS actuarial status θ are less than one for all tested τ values except $\tau = .4$, implying that θ has a negative effect on lifetime utility. This result is somewhat surprising since, in an increasing population economy, the advantage of a pay-as-you-go SS system has been often attributed to the positive effect of θ on lifetime utility. The explanation of our result is obtained through a comparison of the rate of return on physical capital to the implicit rate of return on SS taxes paid. SS tax contributions tends to be dominated by personal saving in terms of the rate of return. More importantly, lifetime utility for some generations increases even with the lack of the dominance.

This matter is further clarified from the simulation results for a range of SS funding policies δ under alternative government fiscal policies regarding the SSTF. In Case I ($a_g = 0$ and $a_k = 1.0$), the only actuarially favorable SS system ($\delta = .8$) yields the second highest steady state value for RWB. In both Case II ($a_g = 1.0$) and III ($a_g = a_k = 0$), regardless of the magnitude of $\overline{RWB}$, the value of θ falls as δ increases. More particularly, in both Case II and III, for a relatively high productivity of human capital ($\lambda = 3.0$), the highest value for $\overline{RWB}$ and the lowest value for θ are realized when the SS system is fully funded ($\delta = 1.0$). This is partly because the SSTF is not validated into equity claim that can provide SS benefits in Case II and III. [Notice that, when $\delta = 1.0$, the value of θ for these cases is 0.]

Overall, the results obtained during the course of our simulations consistently show that SS actuarial status θ cannot be considered to be an effective measure of intergenerational equity, suggesting that an evaluation of a SS system on the basis of SS actuarial status θ can be misleading, especially when the government fiscal use of the SSTF is not considered. It should be noted that the economy at hand responds to SS system in such a way that θ for a given SS system time-invariant.

4.4. SSTF, the Government Fiscal Policy Use of the SSTF, and RWB

When the SSTF is assumed to be invested entirely as physical capital (Case I: $a_g = 0$ and $a_k = 1.0$), the SSTF in period t is capitalized into a claim on capital income in period $t + 1$. As a result, the only effect of a change in the SS funding policy δ lies in how much the change in δ changes the effective SS tax rate $\tilde{\tau}$. As Figure 4.1 shows, the highest value of steady state intergenerational equity as measured by RWB occurs when the SS system is partially funded ($\delta = .6$). This is because a SSTF fraction $\delta = .6$ is the closest to the Pareto optimal SSTF fraction δ^*. However, $\delta = .6$ does not necessarily dominate the other funding policies in terms of RWB in each period. As a matter of fact, given a SS system with $\delta < 1.0$, there is no other SS funding policy δ' such that a switch from δ to δ' increases the value of RWB in every period. This lack of dominance shows how difficult it is to resolve potential conflicts among generations.

When the SSTF is used as redistributive transfers (Case II: $a_g = 1.0$), the SSTF is
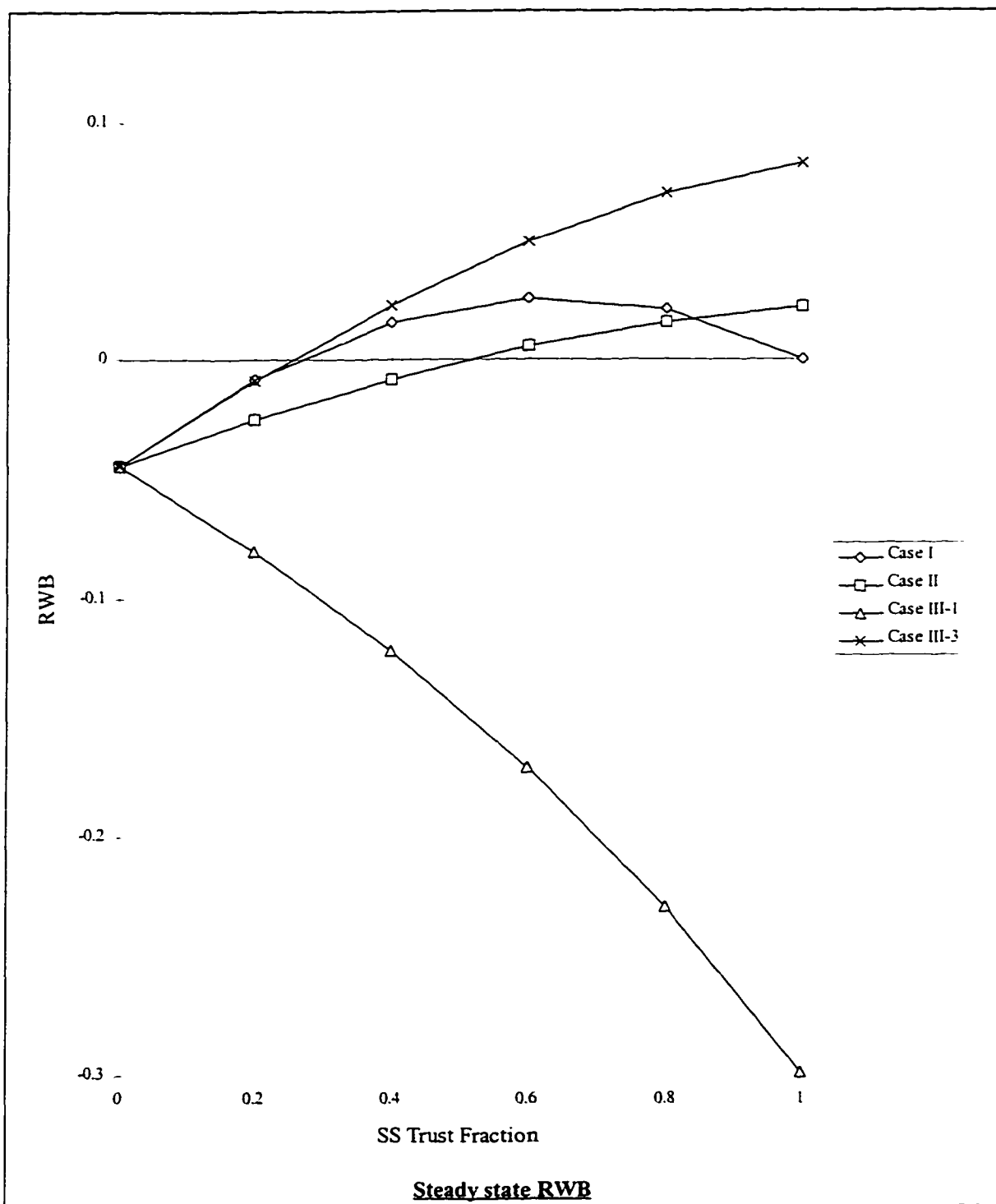
Figure 4.1: :Comparison of intergenerational equity under alternative SS funding policies
δ and alternative government fiscal policy uses of the SSTF
---τ = .2; n = .012, α = .3, β = .5, λ = {1.0, 3.0}

not capitalized into an equity claim for future SS benefits. Thus, agents do not view the SSTF as an other equivalent way to save although a compensating mechanism of SS taxes paid takes place in the form of redistributive transfers. Recall that, due to the absence of a distortion in the labor market, agents in this economic model do not distinguish between different sources of income. Consequently, as long as a certain modified effective SS tax rate $\hat{\tau}$ remains unchanged, a higher SS funding policy δ is equivalent to a lower SS tax rate τ; see (3-4).

As Figure 4.1 shows, the highest value of $\overline{RWB}$ occurs when the SS system is fully funded ($\delta = 1.0$). Based on the modified effective SS tax rate $\hat{\tau}$, we apply the same explanation given in Case I to Case II as well. That is, the steady state rate of return on physical capital closest to the population growth rate occurs when $\delta = 1.0$. A marked possibility of conflict across generations is again borne out in the simulation results of Case II: No SS funding policy δ dominates all other SS funding policies in terms of RWB.

When the SSTF is assumed to be invested entirely as human capital (Case III: $a_g = a_k = 0$), the SSTF is again capitalized but in a different form. Hence, unlike Case I, the SSTF in period t does not directly serve to provide SS benefits for the generation t old in period t + 1. Moreover, unlike Case II, there is no direct compensating mechanism for SS tax contributions that takes place outside the SS system. It is only in terms of changes in the wage rates and the rates of return on physical capital over time that both beneficial

and detrimental effects of alternative possible SS funding policies δ can be tracked down and judged.

Although the two different values for the efficiency parameter λ, ($\lambda = 1.0$ and $\lambda = 3.0$) yield the same steady state physical capital/effective labor ratio, the resulting economic and intergenerational equity consequences are quite different. Clearly, a glimpse of Figure 4.1 is enough to verify this point. When the productivity of human capital investment is relatively low ($\lambda = 1.0$), a higher SS funding policy δ strictly *decreases* the value of $\overline{RWB}$, implying that a less funded SS system is desirable. When the productivity of human capital investment is relatively high ($\lambda = 3.0$), a higher SS funding policy δ strictly *increases* the value of $\overline{RWB}$ (more precisely, RWB in period 3 and later): By this, we infer that, for $\lambda = 3.0$, as δ increases, the beneficial effects of increased wage rates due to enhanced labor skills dominate the negative effects of decreased SS benefits (proportional to the SSTF fraction) and a decrease of capital income (due to decreased rates of return on physical capital).

An important finding from the comparison of these simulation results for Case III is that what really matters for intergenerational equity as measured by RWB is the extent of the benefits (the productivity of the SSTF) and thus human capital technology rather than simply who receives the benefits. This point is further strengthened by recalling that a choice between government physical capital investment and government redistributive transfers is distinguished only by how differently the per-capita physical capital is affected.

There are, however, many potential scenarios which would lead to different intergenerational equity consequences under alternative fiscal policies regarding the SSTF. This is shown by a thought experiment in which a SS system is internally inconsistent in such a way that some kind of financial strain is highly likely to occur. That is, unlike in our economic model, when an agent's SS benefits are assumed to be determined as a fixed fraction of wage income (or disposable income), some kind of financial squeeze is highly likely to occur. In this case the type of government fiscal policy use of the SSTF, together with options taken to resolve the financial deficit, could lead to greatly varying intergenerational equity consequences. In general, this thought experiment illustrates that, when potentially contradicting elements are ingrained together in a SS system, a discussion of intergenerational equity consequences associated with different government uses of the SSTF tend to be inherently complicated.

4.5. Structural Aspects

It is often considered that demographic changes have important economic and intergenerational consequences for a SS system. Yet, our simulation results regarding the sensitivity of $\overline{RWB}$ to a decrease in the net population growth rate show that the relationship between the population growth rate, n, and the SS funding policy δ is weak. In short, the orderings of $\overline{RWB}$ under alternative policies δ associated with two distinctively different population growth rates are shown to be the same, as verified by a comparative dynamic analysis. This finding suggests, for instance, that a particular

demographic structure alone may not make one particular type of SS system preferable to other SS types of systems.

Our simulation results regarding the sensitivity of $\overline{RWB}$ to a decrease in the labor share parameter $(1 - \alpha)$ in the production function and to an increase in the consumption time preference parameter β in the utility function show that $\overline{RWB}$ is highly sensitive to changes in these parameters. Given either a decreased labor share or an increased consumption time preference, agents begin to experience a decrease in steady state intergenerational equity as measured by RWB. Moreover, given either a decreased labor share or increased consumption time preference, a more funded SS system exhibits less of a decrease in $\overline{RWB}$. Overall, our simulation results and sensitivity tests suggest that more funding for a SS system is justified given either a decreased labor share or an increased consumption time preference but not necessarily given a decreased population growth rate.

CHAPTER 5. CONCLUDING REMARKS

As clarified by this study, intergenerational equity depends partly on the type of social security (SS) arrangement and partly on the government fiscal use of the social security trust fund (SSTF). We have been concerned with both aspects, and we have been led to the surprising finding that the particular type of SS arrangement and the government fiscal use of the SSTF may not be particularly important for intergenerational equity when some conditions are satisfied. The chief reasoning for this finding is that, due to the absence of distortion in the labor market, agents tend to be indifferent to the type of saving and the source of income. For instance, when the SSTF is invested in the form of physical capital, agents are indifferent with respect to private versus public saving, and to source of income as long as they attain the same rate of return.

Also, we show in quite a simple way that, although SS actuarial status is one standard measure used in the analysis of SS systems, it is not an effective measure of intergenerational equity. The reason for this is that, although SS actuarial status may affect lifetime utility, it is not the sole factor that does so. Moreover, contrary to the general notion that one of the major underlying reasons requiring the transformation of SS systems is demographic change, we show that having a particular population structure does not necessarily make one type of SS system preferable to other types of SS systems, especially when viewed from the perspective of relative welfare benefit.

We should admit that a careful reading of this dissertation would lead a reader to realize many drawbacks of the current study that should be addressed in future studies. First, quite immediately, a labor economist would find that the current study does not allow for an endogenous labor supply. Many results obtained during the course of our simulations could presumably be modified if, instead, agents were permitted to choose labor supplies, implying that the SS tax rate then has distortionary effects.

Second, the government fiscal use of the SSTF is very primitively specified in the current study. For example, instead of government public consumption expenditure, a simple redistributive transfer expenditure is used to describe the transfer of the SSTF to current living generations. Notice that government transfer expenditures are not included in the traditional gross national product (GNP) at all, hence the current specification may not be the most appropriate way to investigate government fiscal behavior.

Third, the specification of human capital investment is primitive as well. Unlike Lucas (1988) and Romer (1986), the public good aspects of human capital investment, such as increasing returns to scale and spillover effects are not considered. Once a more appropriate human capital production relation is incorporated, the study can be further extended to investigate competing aspects between private and public human capital investments as shown in Orazem and Tesfatsion (1997).

Finally, another great drawbacks in this dissertation is that the economic model postulates a representative agent and thus *intragenerational* equity is completely ignored. However, we do not know yet how to incorporate heterogeneous agents into our

economic model while maintaining the ability to derive explicitly the basic dynamic equations governing the economy. Putting aside this technical difficulty, we expect that the incorporation of *intragenerational* equity into our economic analysis would be immensely fruitful for deepening our general understanding of the term *generational* equity.

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